Revenue Management and Dynamic Pricing[1]

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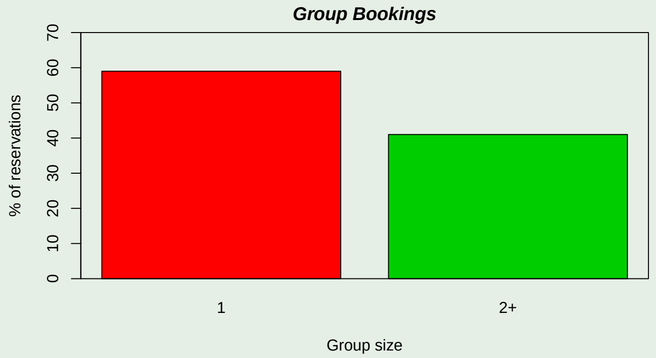
Our Research

- Collaboration with 5 small independent hotels in the Netherlands
- Research motivated by real hotel data

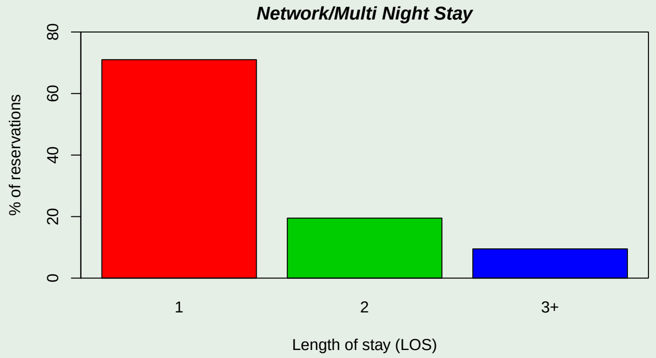
Group Bookings



Observation

Large part (41%) of all bookings are group bookings

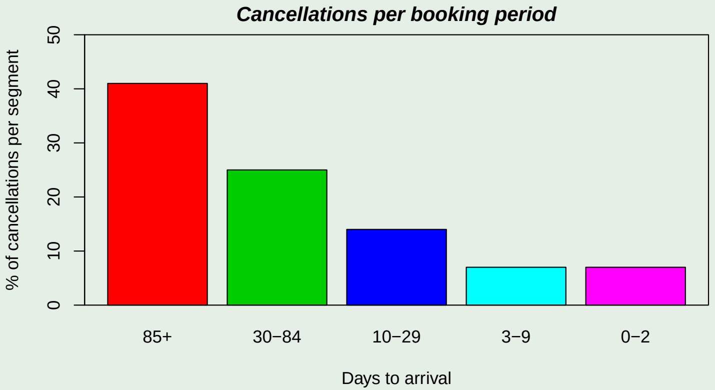
Networks/Multiple Night Stays



Observation

Big part (29%) stays more than one night

Cancellations



Observations

- 22% of all bookings are cancelled
- Early booking $\Longrightarrow$ high cancellation probability

Observations from the Data

- Group bookings (41%)
- Networks (multiple night stays) (29%)
- **Cancellations** (22%)

Customer Choice Cancellation Model

Properties:

- Customer choice behaviour
- Cancellations
- Overbooking

Related work:

- Subramanian et alii (1999): Cancellations
- Talluri and Van Ryzin (2004): Customer choice behaviour

Other Application Areas

Example (Talluri & van Ryzin, 2004)

Hotel with

- $C = 20$ rooms
- $n = 3$ products with prices

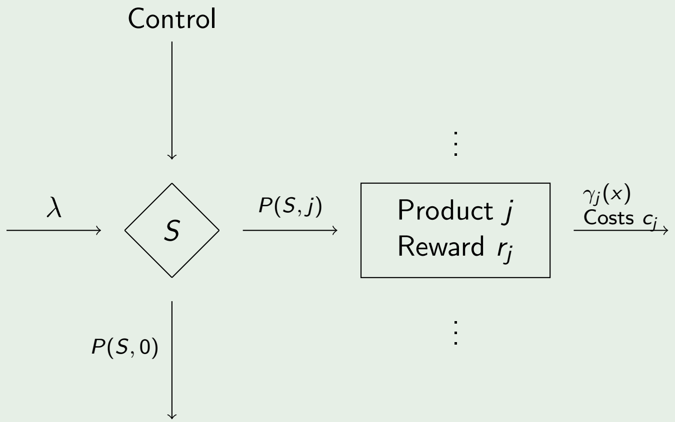
$$r_1 = 160 \qquad r_2 = 100 \qquad r_3 = 90$$

- T days before arrival
- $\lambda = 1/4$ probability that a customer arrives
- x_j number of reservations for product j ($x = (x_1, x_2, x_3)$)
- $\gamma(x_j) = x_j/100$ probability that product j is cancelled
- $c_j = r_j$ costs if product j is cancelled

Example (continued)

- $P(S, j)$ probability that customer buys product j if $S \subset \{1, 2, 3\}$ is offered
- $P(S, 0) = 0$ probability that customer buys nothing
- E.g. $S = \{1, 2\}$ and

$$
\begin{aligned}
P(S,1) &= 0.1 \\
P(S,2) &= 0.6 \\
P(S,3) &= 0 \\
P(S,0) &= 0.3
\end{aligned}
$$


Control
$\vdots$
λ
S
$P(S,j)$
Product j
Reward r_j
$\gamma_j(x)$
Costs c_j
$P(S,0)$
$\vdots$

Objective

Which products $S \subset \{1, 2, 3\}$ do we offer t days before arrival in state x

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Which products $S \subset \{1, 2, 3\}$ do we offer t days before arrival in state x

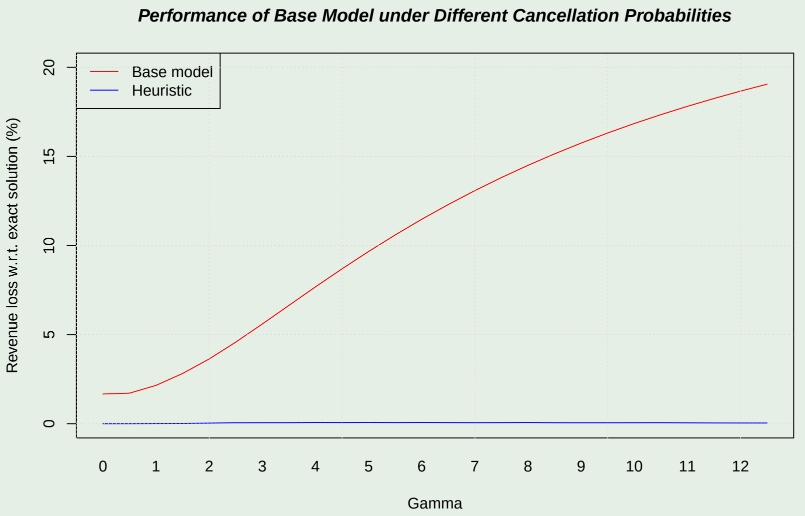
Solution

Model as Markov decision process and solve with dynamic programming:

$$V(x,t) = \max_{S \subset N} \Bigg\{ \lambda \sum_{j \in S} P(S,j)\big(r_j + V(x + e_j, t-1)\big) + \sum_{j=1}^{n} \gamma_j(x)\big(-c_j(t) + V(x - e_j, t-1)\big) + \Bigg(1 - \lambda \sum_{j \in S} P(S,j) - \sum_{j=1}^{n} \gamma_j(x)\Bigg) V(x, t-1) \Bigg\}.$$

Properties

- Reduced state space under assumption $\gamma_j(x) = \gamma x_j$
- Heuristic under this assumption performs well



Performance of Base Model under Different Cancellation Probabilities
Base model
Heuristic
Revenue loss w.r.t. exact solution (%)
0
5
10
15
20
0
1
2
3
4
5
6
7
8
9
10
11
12
Gamma

Estimating Parameters

Maximum Likelihood Function:

$$L(\lambda, \gamma, \beta | x, Z, S, j) = \prod_{t \in D} \left[\lambda P_{tj(t)}(\beta, Z_t, S_t)\right]^{a_\lambda(t)}$$

$$\times \prod_{j=1}^{n} \gamma_j(x_j)^{a_j(t)} \cdot \left[1 - \lambda - \sum_{j=1}^{n} \gamma_j(x_j)\right]^{a(t)}$$

Expectation Maximisation Algorithm

1. Estimate $\hat{a}_\lambda(t)$ and $\hat{a}(t)$
2. Estimate $(\hat{\lambda}, \hat{\beta}, \hat{\gamma})$ using $\hat{a}_\lambda(t)$ and $\hat{a}(t)$
3. Go back to step 1 until stopping criterion

Upside: Converges (eventually) to optimal solution
Downside: Notoriously slow computation time; bad parameter estimation

New Parameter Estimation Algorithm

Based on Newman et alii (2012).

1. Estimate $\hat{\gamma}$
2. Estimate $\hat{\beta}$
3. Estimate $\hat{\alpha}$ and $\hat{\lambda}$ using $\hat{\beta}$

Upside: Significantly faster; better estimates
Downside: Yet to reveal itself

Conclusion

- Cancellations have big impact on revenue
- The heuristic approximates the optimal solution well
- The new parameter estimation method performs well

Further Research

- Application to Dutch hotels
- Expand with group bookings and networks/multiple night stays