Air Cargo Revenue Management: flexible products & robust optimization

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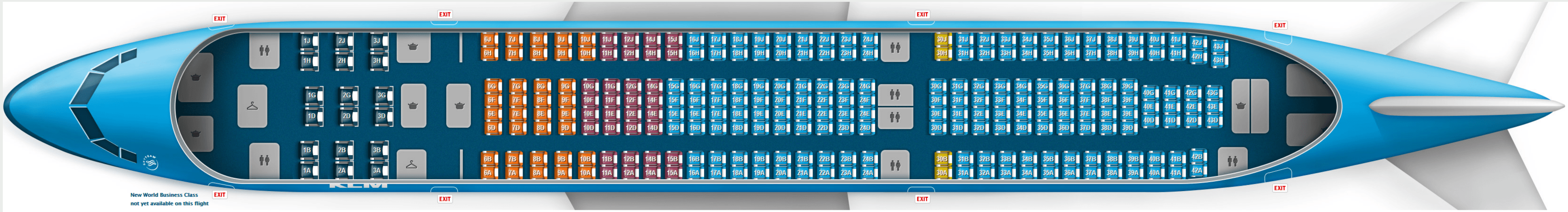
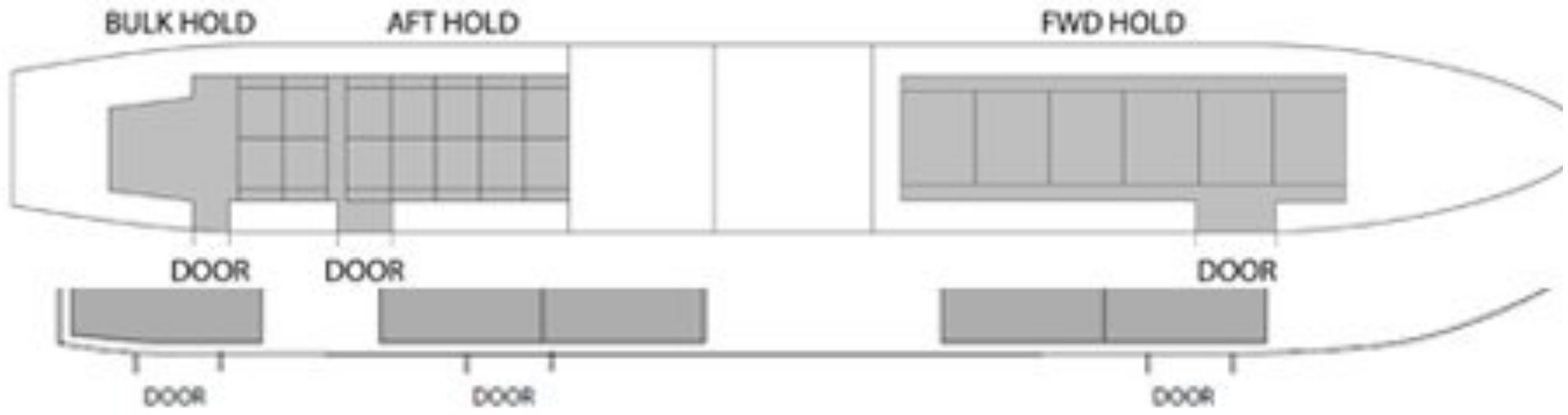
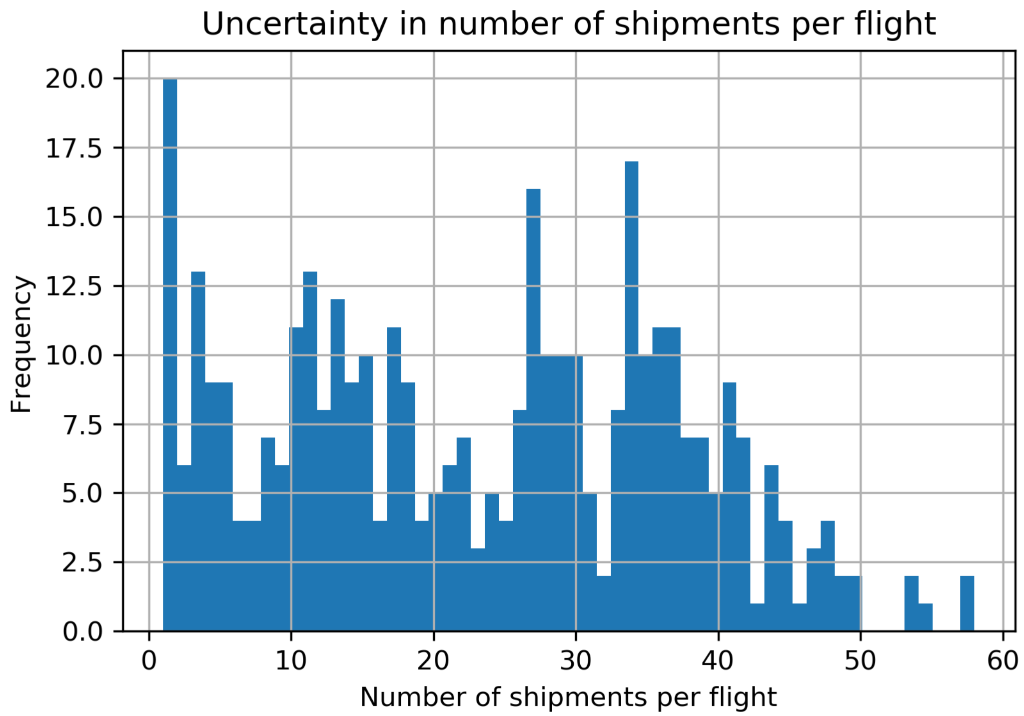
November 2018

Cargo Dynamics: uncertainty in number of shipments (20 Cargo vs 200 PAX)





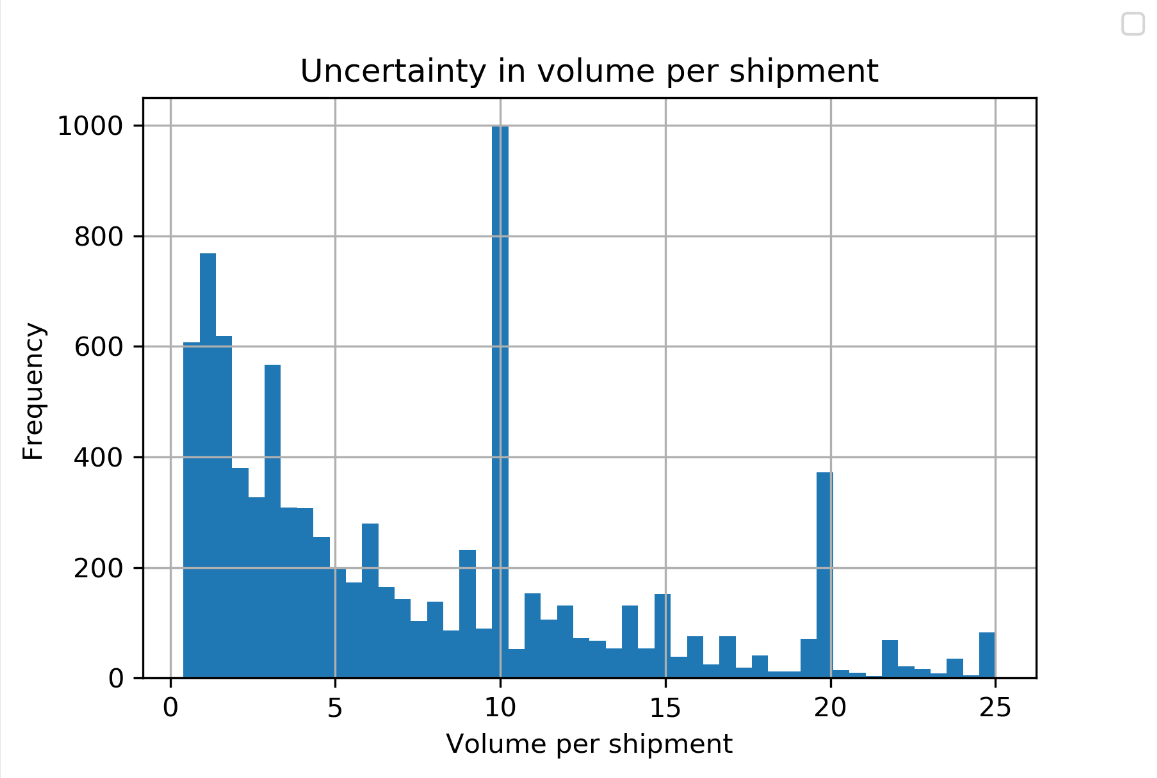


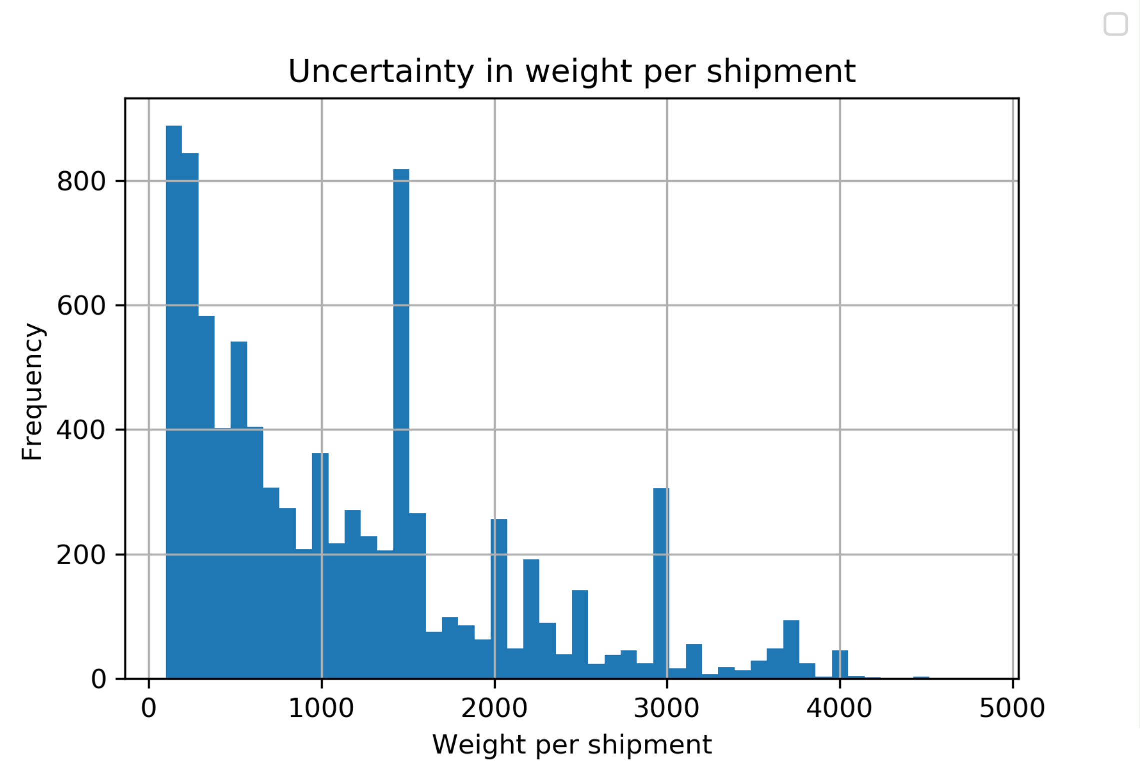
Cargo Dynamics: uncertain capacity

Luggage from passengers consumes potential cargo space

`Last minute' changes to airplane schedules related to PAX optimization

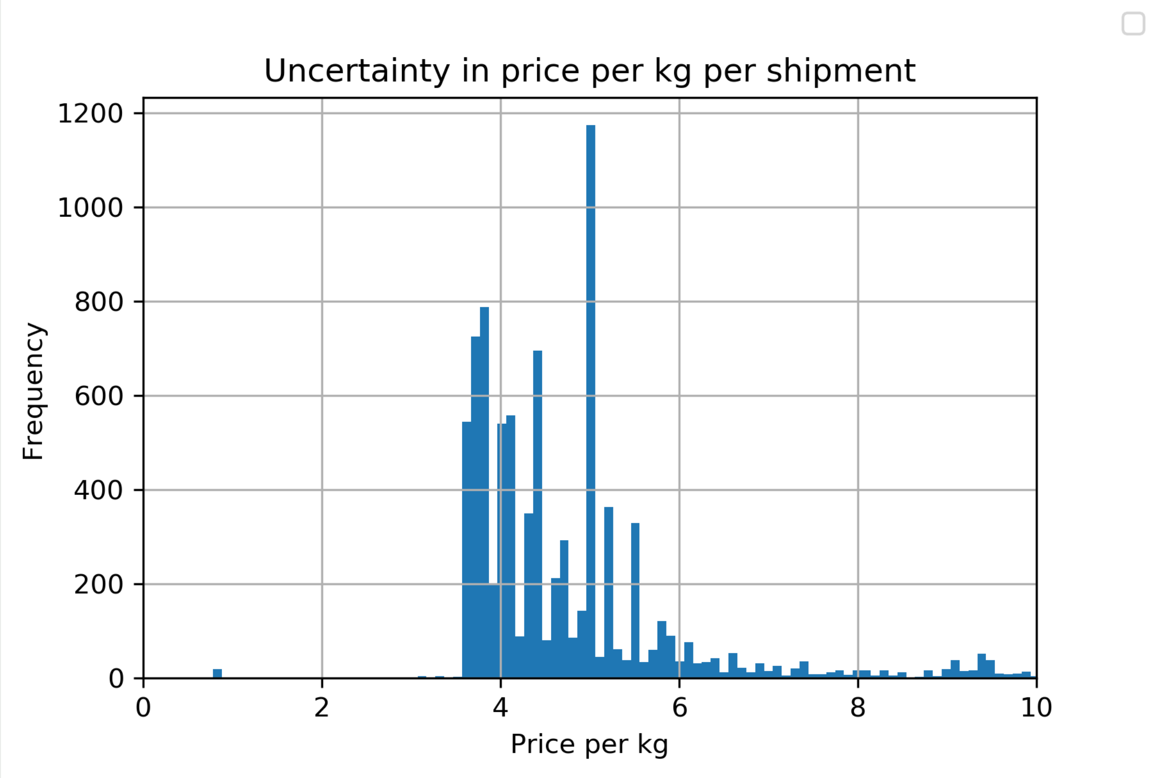
Cargo Dynamics: uncertain volume & weight

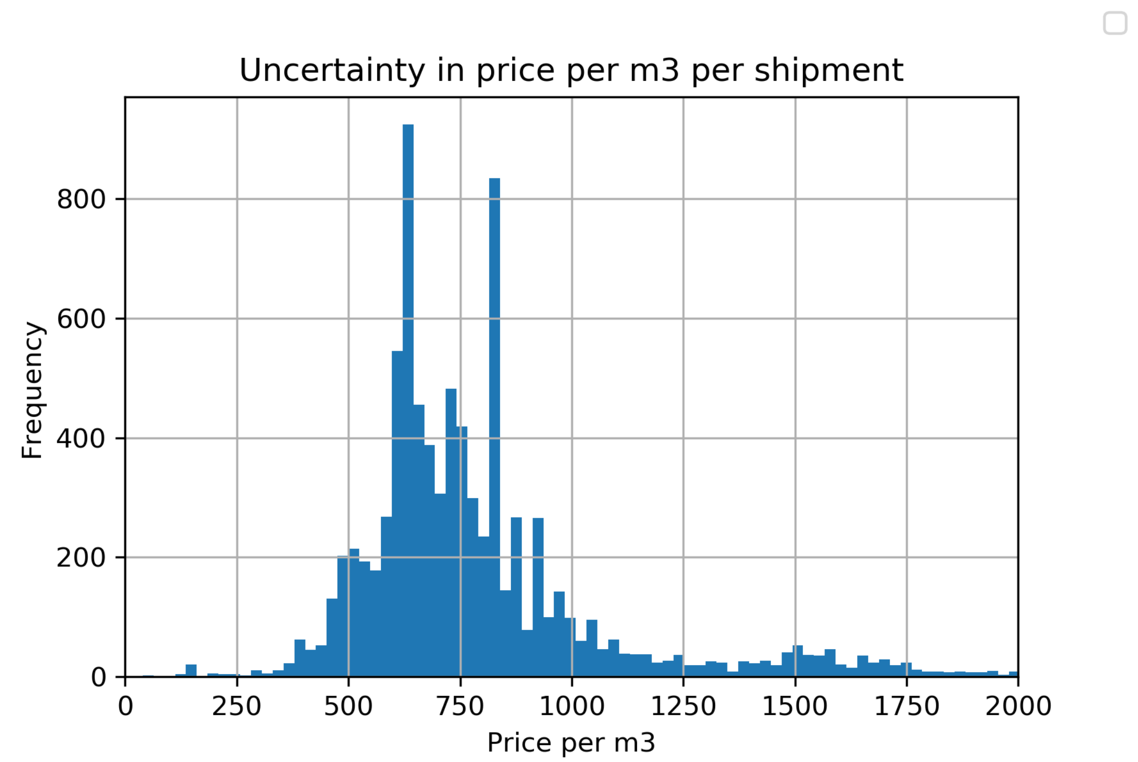




E.g., moving companies do not know exactly how much volume & weight household goods will be consuming

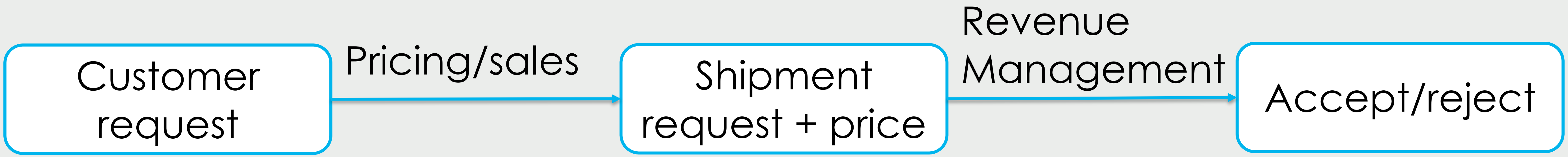
Cargo Dynamics: uncertain price/kg & price/m3





Price/kg and price/m3 vary highly per shipment

RM process in Cargo



RM process in Cargo

In formulas: classic RM case (DLP)

$$\begin{aligned} \max \quad & r^T x \\ s.t. \quad & Ax \leq V \\ & Bx \leq W \\ & 0 \leq x \leq D \end{aligned}$$

objective

volume constraint

weight constraint

Uncertain tender rate

Uncertain capacity

Uncertain demand

In formulas: classic RM case (DLP)

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Solution:

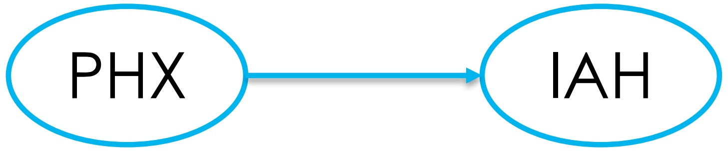
1. Flexible Products
2. Robust Optimization

Flexible Products

A **flexible product** is a *set of alternatives* offered by a firm such that the customer is *assigned* to one of the alternatives at a *later point in time*

Flexible Products Explained

Example network



Flights:

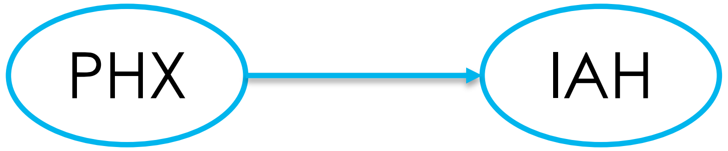
1. **Dep 9:00**
2. **Dep 13:00**

Quotes for 1000kg, 6m3 PHX-IAH request:

1. Dep 9:00: $2,00/Chkg
2. Dep 13:00: $2,00/Chkg

Flexible Products Explained

Example network



Flights:

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Quotes for 1000kg, 6m3 PHX-IAH request:

1. Dep 9:00: $2,00/Chkg
2. Dep 13:00: $2,00/Chkg
3. **Tbd: $1,90/Chkg**

Flexible Products

- Offer quote to ship goods within time window
- Airline assigns flight later on
- Mitigate demand over resources

In formulas: DLP + flexible products

Specific products

Flexible products

$$
\begin{aligned}
\max \quad & \boxed{r^T x} + \boxed{f^T y} && \text{objective}\\
s.t. \quad & Ax + A'z \le V && \text{volume constraint}\\
& Bx + B'z \le W && \text{weight constraint}\\
& y - Uz = 0 && \text{flexible products allocation}\\
& 0 \le x \le D_x\\
& 0 \le y \le D_y
\end{aligned}
$$

Dynamic Programming & Flexible Products – challenges

$$V_t(x_v, x_w, y) = \sum_{j=1}^{n} p_j \max\{r_j + V_{t-1}(x_v - v^j, x_w - w^j, y), V_{t-1}(x_v, x_w, y)\}$$

$$+ \sum_{k=1}^{f} p_k \max\{\rho_k + V_{t-1}(x_v, x_w, y + e_k), V_{t-1}(x_v, x_w, y)\}$$

$$+ (1 - \sum_{j=1}^{n} p_j - \sum_{k=1}^{f} p_k) V_{t-1}(x_v, x_w, y)$$

State space:

$$S = \left\{ (x_v, x_w, y) \middle| \begin{array}{c} x_v + A'z \le V, \\ x_w + B'z \le W, \\ y - Uz = 0 \end{array} \right\},$$

With

$x_v \in \mathbb{R}^m_{\ge 0}$ = available volume per leg $i = 1, \dots, m$

$x_w \in \mathbb{R}^m_{\ge 0}$ = available weight per leg $i = 1, \dots, m$

$y \in \mathbb{N}^f$ = booked flexible shipments

Dynamic Programming & Flexible Products – challenges

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Specific products

Flexible products

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Resource consumption of flexible products may change over time!

Therefore, *traditional* leg-decomposition does not work!

Solution: robust recourse model

$$
\begin{array}{llll}
\max & \sum_{t=1}^{T} r_t^T y^t & & \\
s.t. & Az & \leq V & \\
 & Bz & \leq W & \\
 & \sum_{t=1}^{T} y^t - Uz & = 0 & \\
 & y^t & \leq \bar{D}_t + \zeta_t, & \zeta \in Z \\
 & y^t & \geq 0 &
\end{array}
$$

Recourse modeling: $y = v^t + P_t \zeta v^t$

Let future decision depend on results from the past

Solution: robust recourse model

$$\begin{array}{lll} \max & \sum_{t=1}^{T} \rho_t^T (\nu^t + P_t \zeta \upsilon^t) & \\ s.t. & \sum_{t=1}^{T} A^t (\nu^t + P_t \zeta \upsilon^t) & \leq V \\ & \sum_{t=1}^{T} B^t (\nu^t + P_t \zeta \upsilon^t) & \leq W \\ & \nu^t + P_t \zeta \upsilon^t & \leq D_t + \zeta_t \\ & \nu^t + P_t \zeta \upsilon^t & \geq 0 \end{array}$$

- Accounting for ***stochasticity*** in model
- Allows ***flexible products*** to mitigate demand
- Deals with uncertainty through ***robust optimization***

Thank you

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