Revenue Management under Reviews and Online Ratings

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INFORMS RM&P conference, June 16, 2016

This work is in collaboration with dr. D.M. Roijers
New York City: 19 properties found
3 Reasons to Visit: Central Park, Times Square & The Statue of Liberty

Tip: Open the map to find your perfect place to stay!

- Residence Inn by Marriott New York Downtown Manhattan/World Trade Center Area
  - Neighborhood: Wall Street - Financial District, New York City - Subway Access
  - 14 people are looking right now
  - Booked 12 times today
  - Studio - In high demand - only 5 rooms left on our site!

- Fairfield Inn by Marriott New York Manhattan/Financial District
  - Neighborhood: Wall Street - Financial District, New York City - Subway Access
  - 10 people are looking right now
  - Booked 15 times today
  - Double Room - In high demand - only 5 rooms left on our site!
“Excellent value for money in a great location”

Leisure trip  Couple  King Studio  Stayed 3 nights

- Lifts a little slow at times.
- Great location, right next to subway and downtown sights. Staff helpful and friendly. Room the usual small NY standard, but well appointed and laid out. Felt modern and clean. Bed very comfortable. Breakfast was as expected and always well stocked. Gym was small and more for running / cross training etc (a few dumbbells) but perfectly adequate.

Helpful

“Very clean and modern hotel. Friendly. Near WTC”

Leisure trip  Solo traveler  King Studio  Stayed 2 nights

- Nothing really. Financial district not really where I want to stay but very near subway. I stayed during memorial weekend and was woken by noise in the corridor (just a group getting back) at 5am. Once they were in their rooms (I think they had two rooms next to each other), I couldn’t hear a thing but it all woke me with a start. When there is room (and the rooms here nice and large), I don’t know why designers don’t put in an internal hall, creating an inner hallway. Hotels that do this are great and it blocks off all the noise from the hallway, people coming in/out, discussing what time for breakfast, knocking on friends doors etc. Anyway not the hotel’s fault I stayed during memorial w/e or that I happened to stay next to two rooms who went out partying :-(

Chris
United Kingdom
20 property reviews
2 destination reviews
1 helpful vote
Customers are highly influenced by (negative) information
(Negative) reviews influence customers purchasing products
(Negative) price/quality perception influences customers writing reviews
Trade-off: Revenue vs Rating
Goal: Maximize long-term revenue

- Optimizing revenue might lead to worse ratings and suboptimal revenue
Clients consider the hotel, depending on the reviews $\mathcal{R}$ ($\lambda_i(\mathcal{R})$)

Depending on the products $S$ that are offered, clients make a purchase ($P_j(\mathcal{R}, S)$)

A purchase leads to:
- revenue $r_j$
- positive review probability $q_j^p$
- negative review probability $q_j^n$

Purchased products can be cancelled
\[ \lambda_i(R), P_0(R, S), P_j(R, S) \rightarrow S \rightarrow P_j(R, S) \rightarrow \text{Product } j \rightarrow \gamma_i(R)x_j^i \]

\[ r_j, q_j^p, q_j^n \]
Modelling demand and cancellations as a function of reviews

- $Q^p_i = \#$ positive reviews from day $i$
- $Q^n_i = \#$ negative reviews from day $i$
- $\alpha \in (0, 1)$ discounting parameters
- $M = \#$ past arrival days
- Adjusted reviews:

$$\tilde{Q}_k^p := \sum_{i=k-M}^{k-1} \alpha^{k-i-1} Q^p_i, \quad \tilde{Q}_k^n := \sum_{i=k-M}^{k-1} \alpha^{k-i-1} Q^n_i.$$  

- Rating:

$$\rho := \frac{\tilde{Q}_k^p}{\tilde{Q}_k^p + \tilde{Q}_k^n}.$$
Modelling demand and cancellations as a function of reviews

Demand and cancellation parameters:

\[
\lambda_i(R) := \bar{\lambda}_i \exp (\beta_p^\lambda \rho + \beta_n^\lambda (1 - \rho)),
\]
\[
\gamma_i(R) := \bar{\gamma}_i \exp (\beta_p^\gamma \rho + \beta_n^\gamma (1 - \rho)).
\]
## Multiple arrival day example

<table>
<thead>
<tr>
<th>Day</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
<th>0.6</th>
<th>0.6</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Arrival day 1
- $S_3^1$
- $S_2^1$
- $S_1^1$
- *

### Arrival day 2
- $S_3^2$
- $S_2^2$
- $S_1^2$
- *

### Arrival day 3
- $S_3^3$
- $S_2^3$
- $S_1^3$

<table>
<thead>
<tr>
<th>Day 0</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
</tr>
</thead>
</table>

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Multiple arrival day dynamics

- \( R^i(S^1, \ldots, S^i) \) revenue from arrival day \( i \)
- \( S^i = \{S_T, \ldots, S_1\} \) strategy for arrival day \( i \)
- Objectives:

\[
\phi^I(S) = \sum_{i=1}^{l} R^i(S^1, \ldots, S^i),
\]

\[
\phi^\infty(S) = \sum_{i=1}^{\infty} a^{i-1} R^i(S^1, \ldots, S^i).
\]
Problem
- State space size: $N^I$
- Action space size: $N^T$
- Intractable due to curse of dimensionality

Solution: Equilibrium Solutions
- Keep expected rating constant, equal to a target rating $\rho^*$
- Now $R^i$ depends solely on $S^i$
- Each arrival day can be solved separately
One arrival day Bellman equation

\[ V_{i,t}^\pi(x) = \sum_{S \subseteq N} \pi(x, t, S) \left\{ \lambda \sum_{j \in S} P_j(S) [r_j^i - \Delta H_j^i(t) + V_{i,t-1}^\pi(x + 1)] + \gamma x V_{i,t-1}^\pi(x - 1) \right. \]

\[ + \left. \left( 1 - \lambda \sum_{j \in S} P_j(S) - \gamma x \right) V_{i,t-1}^\pi(x) \right\}. \]

Three objectives:

\[ \mathbf{V}^\pi = (V_1^\pi, V_2^\pi, V_3^\pi) \]

Scalarization:

\[ V_w^\pi = f(\mathbf{V}^\pi, \mathbf{w}) = w_1 V_1^\pi + w_2 V_2^\pi + w_3 V_3^\pi \]
Convex Coverage Set (CCS)

Every weight $w$ has a corresponding optimal policy $\pi$ and value function $V^\pi$.
Finding equilibrium solution

- Policy $\pi \in CCS$ corresponds to value vector $V^\pi$
- Hyperplane where rating $= \rho$:

$$\mathcal{H} = \left\{ x \in \mathbb{R}^3 \left| \frac{x_2}{x_2 - x_3} = \rho \right. \right\} = \left\{ x \in \mathbb{R}^3 \left| x_2(\rho - 1) - \rho x_3 = 0 \right. \right\}.$$ 

- Target-rating policy:

$$\pi(\rho) = \arg \max_{\pi \in CCS} \left\{ V_1^\pi \left| V^\pi \in CCS' \cap \mathcal{H} \right. \right\}$$

- Optimal target-rating and corresponding policy:

$$\pi(\rho^*) = \arg \max_{\pi(\rho)} V_1^{\pi(\rho)}$$
## One instance: policy analysis

<table>
<thead>
<tr>
<th>Revenue</th>
<th>Positive reviews</th>
<th>Negative reviews</th>
<th>Rating $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>7493.76</td>
<td>33.01</td>
<td>3.75</td>
<td>0.90</td>
</tr>
<tr>
<td>12513.99</td>
<td>42.88</td>
<td>9.28</td>
<td>0.82</td>
</tr>
<tr>
<td>12595.63</td>
<td>41.42</td>
<td>6.30</td>
<td>0.87</td>
</tr>
<tr>
<td>14252.80</td>
<td>42.48</td>
<td>9.77</td>
<td>0.81</td>
</tr>
<tr>
<td>15137.33</td>
<td>41.09</td>
<td>7.57</td>
<td>0.84</td>
</tr>
<tr>
<td>16021.48</td>
<td>41.67</td>
<td>10.38</td>
<td>0.80</td>
</tr>
<tr>
<td>17983.01</td>
<td>40.40</td>
<td>11.16</td>
<td>0.78</td>
</tr>
<tr>
<td>18693.24</td>
<td>39.78</td>
<td>10.61</td>
<td>0.79</td>
</tr>
<tr>
<td>19985.47</td>
<td>38.61</td>
<td>9.99</td>
<td>0.79</td>
</tr>
<tr>
<td>21719.66</td>
<td>36.79</td>
<td>11.74</td>
<td>0.76</td>
</tr>
<tr>
<td>23528.79</td>
<td>33.55</td>
<td>12.81</td>
<td>0.72</td>
</tr>
<tr>
<td>24686.86</td>
<td>26.26</td>
<td>12.34</td>
<td>0.68</td>
</tr>
<tr>
<td>25194.50</td>
<td>29.93</td>
<td>13.87</td>
<td>0.68</td>
</tr>
<tr>
<td>26943.83</td>
<td>25.33</td>
<td>15.07</td>
<td>0.63</td>
</tr>
<tr>
<td>27098.09</td>
<td>21.11</td>
<td>15.62</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Observations

1. By sacrificing revenue the rating can be increased
2. Sacrificing revenue not always increases rating
3. When revenue increases, positive reviews increase and negative reviews decrease
4. However, increases are not strict due to the trade-off
1. Large effects of demand and review probabilities
2. Large effect of demand, small effect of review probabilities
3. Small effect of demand, large effect of review probabilities
4. Small effects of demand and review probabilities
Observations

1. All scenarios show structural increase in revenue, of up to 11%.
2. 11.1% increase in rating leads to 5.7% increase in revenue (similar to Ye et alii (2011))
Implications

- Tractable solution methods
- Improving hotel facilities
- Multiple night stay