Choice-based Network Revenue Management under Reviews

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INFORMS annual, Nashville, Tennessee, USA, Nov 15, 2016
<table>
<thead>
<tr>
<th>Name</th>
<th>Rating</th>
<th>Review</th>
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<tbody>
<tr>
<td>Mark</td>
<td>10</td>
<td><strong>“Excellent value for money in a great location”</strong></td>
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<tr>
<td></td>
<td></td>
<td>- Leisure trip</td>
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<td></td>
<td></td>
<td>Lfts a little slow at times.</td>
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<td></td>
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<td>Great location, right next to subway and downtown sights. Staff helpful and friendly. Room the usual small NY standard, but well appointed and laid out. Felt modern and clean. Bed very comfortable. Breakfast was as expected and always well stocked. Gym was small and more for running / cross training etc (a few dumbbells) but perfectly adequate.</td>
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<tr>
<td>Chris</td>
<td>9.2</td>
<td><strong>“Very clean and modern hotel. Friendly. Near WTC”</strong></td>
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<tr>
<td></td>
<td></td>
<td>- Leisure trip</td>
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<td>Nothing really. Financial district not really where I want to stay but very near subway. I stayed during memorial weekend and was woken by noise in the corridor (just a group getting back) at 5am. Once they were in their rooms (I think they had two rooms next to each other), I couldn’t hear a thing but it all woke me with a start. When there is room (and the rooms here nice and large), I don’t know why designers don’t put in an internal hall, creating an inner hallway. Hotels that do this are great and it blocks off all the noise from the hallway, people coming in/out, discussing what time for breakfast, knocking on friends doors etc. Anyway not the hotel’s fault I stayed during memorial w/e or that I happened to stay next to two rooms who went out partying :-(</td>
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Customers are highly influenced by (negative) information
(Negative) reviews influence customers purchasing products
(Negative) price/quality perception influences customers writing reviews
Trade-off: Revenue vs Rating
Goal: Maximize long-term revenue

- Optimizing revenue might lead to worse ratings and suboptimal revenue
Control

\[ \lambda_d(R) \]

\[ S \]

\[ P_j(R, S) \]

Product \( j \)

\[ r_j, q_j^p, q_j^n \]

\[ P_0(R, S) \]
$d$: moments when reviews are released
Modelling demand as a function of reviews

Demand in period $d$:

$$\lambda_d := (\bar{\lambda}_d + \beta^p \tilde{Q}^p_{d-1} + \beta^n \tilde{Q}^p_{d-1}),$$

where

- $\tilde{Q}^p_d = \#$ discounted positive reviews from period $d$
- $\tilde{Q}^n_d = \#$ discounted negative reviews from period $d$
- $\beta^p, \beta^n \in \mathbb{R}$ review attribute parameters
Stochastic Problem Formulation

The problem statement is given by

$$\max_{\pi \in \Pi} \mathbb{E}\left[ \int_0^T r^\top N(S_\pi(t)) \, dt \right]$$

expected revenue

s.t. \( \int_0^T A N(S_\pi(t)) \leq C \),

capacity constraint

$$S_\pi(t) \subset N,$$

where

\( \Pi \) policy space

\( N(S_\pi(t)) \in \mathbb{N}^n \) stochastic process of the vector of purchases at time \( t \) under policy \( \pi \)

\( S_\pi(t) \subset N \) offer set corresponding to \( \pi \)

\( A = (a_{ij}) \) resource consumption matrix

\( C \in \mathbb{N}^m \) resource capacity vector

\( r \in \mathbb{N}^n \) reward vector
Stochastic Problem: Limitations

- High dimensional state space (resources and reviews)
- Large number of products and resources

Solution $\rightarrow$ deterministic demand and reviews
The choice-based review linear program (CRLP) is given by

$$\max_{x(S,d): S \subset N, 1 \leq d \leq D} \sum_{d=1}^{D} \sum_{S \subset N} x(S,d) \sum_{j \in S} P_j(S)r_j$$

subject to

$$\sum_{d=1}^{D} \sum_{S \subset N} x(S,d)AP(S) \leq C,$$

$$\sum_{S \subset N} x(S,d) \leq \left( \tilde{\lambda}_d + \sum_{S \subset N} \sum_{d'=1}^{d-1} x(S,d')\mu(S,d,d') \right)^+.$$ 

- $x(S,d)$ arrivals when $S$ is offered in period $d$ (decision variables)
- $\mu(S,d,d')$ impact of arrivals from period $d'$ when set $S$ is offered on demand in period $d$
Analytical Results

**Proposition 1: Upper bound**

\[ V^{CRLP} \geq V^* \]

**Proposition 2: asymptotic optimality**

\[
\lim_{k \to \infty} \frac{1}{k} V^*_k = \lim_{k \to \infty} \frac{1}{k} V^{CRLP}_k = V^{CRLP}.
\]

- \( V^{CRLP} \): optimal revenue from CRLP
- \( V^* \): optimal expected revenue from stochastic problem
- \( V^{CRLP}_k \): revenue for \( k \)-scaled CRLP
- \( V^*_k \): revenue for \( k \)-scaled stochastic problem
Solution Methods

CRLP
Offer set $S$ during period $d$ according to $x(S, d)$

Robust CRLP
Motivation: outcome of reviews is uncertain, but impacts future demand
Consider demand constraint of CRLP:

\[
\sum_{S \subset N} x(S, d) \leq \left( \tilde{\lambda}_d + \sum_{S \subset N} \sum_{d' = 1}^{d-1} x(S, d') \mu(S, d, d') \right)^+.
\]

Assume uncertainty in \( \mu(S, d, d') \):

\[
\mu(S, d, d') = \overline{\mu}(S, d, d') + \zeta(S, d, d'),
\]

where \( \zeta \in Z \), with \( Z \) an uncertainty region of \( \zeta \).
Consider an adaptation of box/interval uncertainty:

\[ Z = \{ \zeta \mid |\zeta(S, d, d')| \leq \rho(S, d, d') \} , \]

with \( \rho(S, d, d') > 0 \). The constraint can be rewritten to:

\[
\sum_{S \subseteq N} x(S, d) \leq \left( \tilde{\lambda}_d + \sum_{S \subseteq N} \sum_{d' = 1}^{d-1} x(S, d') [\bar{\mu}(S, d, d') - \rho(S, d, d')] \right)^+ .
\]
Robust CRLP

Upside:
- Leads to tractable MILP (no extra constraints)
- $\rho(S, d, d')$ can be set relatively to the nominal value $\bar{\mu}(S, d, d')$
- (With box uncertainty $\rho(S, d, d') = \rho$, independent from $S$, $d$, and $d'$)

Downside:
- Conservative
- Compare strategies from CRLP, CDLP (benchmark), and Robust CRLP
- Simulate to get expected revenue $\mu$ and standard deviation $\sigma$
- Optimality gap to measure **expected revenue**
- Coefficient of variation ($c_v = \sigma/\mu$) to measure **robustness**