Air Cargo Revenue Management: flexible products & robust optimization

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Cargo Dynamics: uncertainty in number of shipments (20 Cargo vs 200 PAX)
Cargo Dynamics: uncertain capacity

Luggage from passengers consumes potential cargo space

`Last minute’ changes to airplane schedules related to PAX optimization
Cargo Dynamics: uncertain volume & weight

E.g., moving companies do not know exactly how much volume & weight household goods will be consuming.
Cargo Dynamics: uncertain price/kg & price/m3

Price/kg and price/m3 vary highly per shipment
RM process in Cargo

1. Customer request
2. Pricing/sales
3. Shipment request + price
4. Revenue Management
5. Accept/reject
RM process in Cargo

Input/given

Customer request → Pricing/sales → Shipment request + price

Goal/objective

Revenue Management
Accept/reject
In formulas: classic RM case (DLP)

\[
\begin{align*}
\text{max} & \quad r^T x & \text{objective} \\
\text{s.t.} & \quad Ax \leq V & \text{volume constraint} \\
& \quad Bx \leq W & \text{weight constraint} \\
& \quad 0 \leq x \leq D
\end{align*}
\]

Uncertain tender rate
Uncertain capacity
Uncertain demand
In formulas: classic RM case (DLP)

\[
\begin{align*}
\max & \quad r^T x \\
\text{s.t.} & \quad Ax \leq V \\
& \quad Bx \leq W \\
& \quad 0 \leq x \leq D
\end{align*}
\]

Uncertain tender rate
Uncertain capacity
Uncertain demand

Solution:
1. Flexible Products
2. Robust Optimization
Flexible Products

A **flexible product** is a set of alternatives offered by a firm such that the customer is **assigned** to one of the alternatives at a **later point in time**
Flexible Products Explained

Example network

```
PHX ---- IAH
```

Flights:
1. Dep 9:00
2. Dep 13:00

Quotes for 1000kg, 6m³ PHX-IAH request:
1. Dep 9:00: $2,00/Chkg
2. Dep 13:00: $2,00/Chkg
Flexible Products Explained

Example network

PHX  ----> IAH

Flights:
1. Dep 9:00
2. Dep 13:00

Quotes for 1000kg, 6m3 PHX-IAH request:
1. Dep 9:00:
   $2,00/Chkg
2. Dep 13:00:
   $2,00/Chkg
3. Tbd:
   $1,90/Chkg

Flexible Products

- Offer quote to ship goods within time window
- Airline assigns flight later on
- Mitigate demand over resources
In formulas: DLP + flexible products

\[ \max \quad r^T x + f^T y \]

\[ \text{s.t.} \quad Ax + A'z \leq V \quad \text{volume constraint} \]
\[ Bx + B'z \leq W \quad \text{weight constraint} \]
\[ y - Uz = 0 \quad \text{flexible products allocation} \]
\[ 0 \leq x \leq D_x \]
\[ 0 \leq y \leq D_y \]
Dynamic Programming & Flexible Products – challenges

\[ V_t(x_v, x_w, y) = \sum_{j=1}^{n} p_j \max \{r_j + V_{t-1}(x_v - v^j, x_w - w^j, y), V_{t-1}(x_v, x_w, y) \} \]

\[ + \sum_{k=1}^{f} p_k \max \{\rho_k + V_{t-1}(x_v, x_w, y + e_k), V_{t-1}(x_v, x_w, y) \} \]

\[ + (1 - \sum_{j=1}^{n} p_j - \sum_{k=1}^{f} p_k) V_{t-1}(x_v, x_w, y) \]

State space:

\[ S = \left\{ (x_v, x_w, y) \mid \begin{array}{l}
  x_v + A'z \leq V, \\
  x_w + B'z \leq W, \\
  y - Uz = 0
\end{array} \right\} \]

With

\[ x_v \in \mathbb{R}_{\geq 0}^m = \text{available volume per leg } i = 1, \ldots, m \]

\[ x_w \in \mathbb{R}_{\geq 0}^m = \text{available weight per leg } i = 1, \ldots, m \]

\[ y \in \mathbb{N}^f = \text{booked flexible shipments} \]
Dynamic Programming & Flexible Products – challenges

\[
V_t(x_v, x_w, y) = \sum_{j=1}^{n} p_j \max \{ r_j + V_{t-1}(x_v - v^j, x_w - w^j, y), V_{t-1}(x_v, x_w, y) \}
+ \sum_{k=1}^{f} p_k \max \{ \rho_k + V_{t-1}(x_v, x_w, y + e^k), V_{t-1}(x_v, x_w, y) \}
+ (1 - \sum_{j=1}^{n} p_j - \sum_{k=1}^{f} p_k) V_{t-1}(x_v, x_w, y)
\]

State space:

\[
S = \left\{ (x_v, x_w, y) \left| \begin{array}{l}
    x_v + A' z \leq V, \\
    x_w + B' z \leq W, \\
    y - U z = 0
\end{array} \right. \right\}
\]

With

- \( x_v \in \mathbb{R}_{\geq 0}^m = \) available volume per leg \( i = 1, \ldots, m \)
- \( x_w \in \mathbb{R}_{\geq 0}^m = \) available weight per leg \( i = 1, \ldots, m \)
- \( y \in \mathbb{N}^f = \) booked flexible shipments
$$V_t(x_v, x_w, y) = \sum_{j=1}^{n} p_j \max \{ r_j + V_{t-1}(x_v - v^j, x_w - w^j, y), V_{t-1}(x_v, x_w, y) \}$$

$$+ \sum_{k=1}^{f} p_k \max \{ \rho_k + V_{t-1}(x_v, x_w, y + e_k), V_{t-1}(x_v, x_w, y) \}$$

$$+ (1 - \sum_{j=1}^{n} p_j - \sum_{k=1}^{f} p_k)V_{t-1}(x_v, x_w, y)$$

State space:

$$S = \left\{ (x_v, x_w, y) \mid \begin{array}{l}
    x_v + A'z \leq V, \\
    x_w + B'z \leq W, \\
    y - Uz = 0
\end{array} \right\}.$$
Solution: robust recourse model

\[
\begin{align*}
\text{max} & \quad \sum_{t=1}^{T} r_t^T y^t \\
\text{s.t.} & \quad Az \leq V \\
& \quad Bz \leq W \\
& \quad \sum_{t=1}^{T} y^t - Uz = 0 \\
& \quad y^t \leq \bar{D}_t + \zeta_t, \quad \zeta \in Z \\
& \quad y^t \geq 0
\end{align*}
\]

Recourse modeling: \( y = v^t + P_t \zeta v^t \)
Let future decision depend on results from the past
Solution: robust recourse model

\[
\begin{align*}
\text{max} \quad & \sum_{t=1}^{T} \rho_t^T (\nu^t + P_t \zeta v^t) \\
\text{s.t.} \quad & \sum_{t=1}^{T} A^t (\nu^t + P_t \zeta v^t) \leq V \\
& \sum_{t=1}^{T} B^t (\nu^t + P_t \zeta v^t) \leq W \\
& \nu^t + P_t \zeta v^t \leq D_t + \zeta_t \\
& \nu^t + P_t \zeta v^t \geq 0
\end{align*}
\]

• Accounting for **stochasticity** in model
• Allows **flexible products** to mitigate demand
• Deals with uncertainty through **robust optimization**
Thank you