Choice-Based Network Revenue Management under Online Reviews

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Abstract

This paper proposes a choice-based network revenue management model that integrates the effect of reviews. The dependency between reviews and revenue is two-fold: customers write reviews based on their price/quality perception, and reviews impact sales. A complicating factor in this model is that the effects of reviews are delayed, e.g., by sacrificing revenue now in order to get better reviews, long-term revenue can be increased. Faced by the complexity of the model, two heuristics are proposed, one of which uses robust optimisation techniques. Numerical results show a 3.5-5.2 percent improvement when reviews are taken into account. Moreover, the impact of reviews is greater under low demand intensity than under high demand intensity.

Keywords: network revenue management, online reviews, choice behaviour, mixed integer linear programming

1 Introduction

A recent development that impacts sales of numerous industries is that customers are given the opportunity to share their experience with other potential clients via websites like Booking.com, Expedia, or Tripadvisor. Examples of industries where this is common practice nowadays are hotels, airlines, travel agencies, car rentals, and short-term storage space leases. Evidence from literature shows that customers are highly influenced by reviews in their purchasing process [Pan and Zhang, 2011, Park and Lee, 2009, Yoo and Gretzel, 2011]. In particular, negative information has relatively more impact than positive information [Sparks and Browning, 2011]; positive reviews improve the confidence and willingness to buy at a hotel [Vermeulen and Seegers, 2009]; and more recent reviews have more impact than older reviews [Pavlou and Dimoka, 2006, Vermeulen and Seegers, 2009, Ye et al., 2011]. In order to increase demand it is therefore beneficial for a company to get more positive reviews and try to avoid negative reviews. On the other hand, the price/quality perception of the customer impacts the reviews. This paper proposes a decision model that maximises revenue in the long run in a network set-up, where reviews are implicitly incorporated.

This pioneering paper studies the problem of deciding which products should be offered at what time, with the objective of maximising revenue, in a network setting where the demand depends on past reviews and customer choice preferences are taken into account. In particular, the reviews are modelled as a feedback mechanism: on the one hand demand depends on reviews, and on the other hand, reviews depend on the price/quality perception of customers. This feedback mechanism was introduced by [Sierag and Roijers, 2016], who consider a *single-leg* choice-based RM problem under reviews. In line with [Sierag and Roijers, 2016], the focus is on reviews rather than ratings, since recent reviews impact sales and ratings tend to remain more or less constant. Demand of future arrivals in the planning horizon depends on past reviews as well as reviews that are released during the planning horizon. This highly complex stochastic problem is intractable, and this paper proposes two heuristics to approximate the problem. The first heuristic is a deterministic variant of the problem, which is shown to be an upper bound to the stochastic problem and converges to the optimal value when demand and capacity are scaled. The second heuristic uses robust optimisation techniques to deal with uncertainty in reviews. Numerical results in Section 5 show that taking reviews into account leads to a revenue improvement of 3.5-5.2 percent. Moreover, the results show that 1) reviews have more impact when the demand is low; and 2) that small hotels are more effected by the review mechanism than larger hotels.

This paper builds on two research areas. The first is the work on choice-based network RM models. The seminal studies by [Gallego et al., 2004] and [Liu and van Ryzin, 2008] provide detailed analysis of such problems. The full stochastic problems proposed in these papers are intractable, so they provide some heuristics based on the deterministic variant of the problem, often referred to as *choice-based deterministic linear program (CDLP)*. The authors of [Kunnumkal and Topaloglu, 2008] provide a variant of CDLP that numerically gives tighter upper bounds and higher expected revenue. The authors of [Talluri, 2014] propose the segment-based deterministic concave program (SDCP), based on segments and their consideration sets. SDCP is a relaxation of CDLP, which is easier to solve, and therefore gives looser upper bounds, but coincides with CDLP when segments have disjoint consideration sets. The authors of [Meissner et al., 2013] provide solution methods for general choice models that approximate CDLP quite efficiently in terms of computation time. Recently, the authors of [Vossen and Zhang, 2015] proposed a dynamic disaggregation method solve CDLP, using column and constraint generations techniques.
The second research area is on e-word-of-mouth (eWOM), in particular the effect of reviews on demand and the effect of sales on reviews. See for example the survey by [Ye et al., 2011] for an overview of related literature. Closest to the work in this paper is our earlier work in [Sierag and Roijers, 2016], where we propose a similar feedback mechanism to deal with reviews in a single-leg environment. The solution methods of [Sierag and Roijers, 2016] cannot be extended to a network setting, and therefore this study is essential.

This paper makes the following research contributions: 1) a model that incorporates reviews in the optimisation process in a network setting (Section 2); 2) a deterministic variant of the intractable stochastic problem, which is shown to be an upper bound and converges asymptotically to the optimal revenue (Section 3); 3) Two heuristics to approximate the stochastic problem: one based on the deterministic problem and one based on robust optimisation methods; 4) numerical experiments indicating that including the review feedback mechanism lead to higher long-term revenue (Section 5). Finally, implications for research and practice are discussed in Section 6.

2 Stochastic Model

This section introduces the full stochastic problem of optimising long-term revenue in a network setting under customer choice behaviour and reviews. For clarity of presentation, this paper is written in the context of hotels, since the combination of reviews and the network structure, in the form of multiple night stays, is very typical for this branch. However, the models and analysis are more general and apply to all network revenue management problems where demand depends on reviews and reviews depend on the experience of the customer. See also Remark 1.

2.1 Model Description

Consider a hotel with identical rooms. The hotel manager wants to sell the rooms for $m \in \mathbb{N}$ nights. The capacity C_i for night $i \in \{1, \ldots, m\}$ is allowed to differ per night, for example due to renovation of some rooms. The firm sells the rooms in the form of n *products*, where product $j \in N = \{1, \ldots, n\}$ is a combination of one or more rooms, possibly for multiple nights, a reward r_j, and certain conditions (like a cancellation policy). In the hotel context used in this paper, the room consumption is determined by an arrival night i and a length of stay (LOS). The reward r_j may include the expected revenue from the room price and other sources of revenue, such as food and beverage, spa and fitness, and casino revenues. Let $A = (a_{ij}) \in \mathbb{R}^{m \times n}$ be the incidence matrix of the products and rooms, where a_{ij} is the amount of rooms of night i that is consumed by product j. Furthermore, a customer that purchased product j has a probability of q_j^p that he will write a positive review about its purchase and an independent probability of q_j^n that he will write a negative review. Hence he will write both a positive and negative review with probability $q_j^p q_j^n$. Parameters are chosen such that $q_j^p + q_j^n + q_j^p q_j^n \leq 1$. Note that in practice it is natural that customers write both positive and negative reviews at the same time, as a customer can write about both positive and negative aspects of the experience.

The products are sold continuously over T time units. Arrival nights occur also within the booking horizon, so products may perish before the end of the booking horizon is met. At certain moments in time T_d ($1 \leq d \leq D$, $D \in \mathbb{N}$) the reviews are updated: all products for which consumption ended after T_{d-1} but before or at T_d, denoted by the set $N_d \subset N$, are published at time T_d. The period between T_{d-1} and T_d is denoted by *period* d. Let Q_d^p and Q_d^n be the number of positive reviews and negative reviews, respectively, that are published at time T_d (with $T_0 = 0$, the start of the booking horizon). The initial positive and negative reviews are given by Q_0^p and Q_0^n, respectively.

Demand is influenced by reviews, either positive or negative [Pan and Zhang, 2011, Park and Lee, 2009, Sparks and Browning, 2011, Vermeulen and Seegers, 2009, Yoo and Gretzel, 2011]. More recent reviews have more impact on demand than older reviews [Pavlou and Dimoka, 2006, Vermeulen and Seegers, 2009, Ye et al., 2011]. To capture this effect, in accordance with the work by [Sierag and Roijers, 2016], the positive and negative reviews are discounted by a factor $\alpha \in (0, 1)$. The *discounted reviews* for time T_d are given by

$$\tilde{Q}_d^p := \sum_{d'=0}^{d} \alpha^{T_d - T_{d'}} Q_{d'}^p, \qquad \tilde{Q}_d^n := \sum_{d'=0}^{d} \alpha^{T_d - T_{d'}} Q_{d'}^n. \tag{1}$$

Assume that customers in period d arrive according to a Poisson process with rate λ_d, where λ_d depends on the the reviews:[1]

$$\lambda_d = (\bar{\lambda}_d + \beta^p \tilde{Q}^p_{d-1} + \beta^n \tilde{Q}^n_{d-1})^+, \tag{2}$$

where $\bar{\lambda}_d \in \mathbb{R}$ is the base arrival rate and $\beta^p, \beta^n \in \mathbb{R}$. The operator $(x)^+$ is defined by $(x)^+ = \max\{0, x\}$, which enforces demand to be non-negative. In this set-up, the LOS and arrival night are included in the product $j \in N$. In accordance with [Vermeulen and Seegers, 2009] it is assumed that positive reviews have positive effect on demand and negative reviews have a negative effect on demand, which translates to $\beta^p_d > 0$ and $\beta^n_d < 0$. In line with findings in literature [Sparks and Browning, 2011], the effect of negative reviews is greater than positive reviews: $\beta^p_d < -\beta^n_d$.

The results of this paper can be derived when the parameters β^p, β^n depend on time, but for clarity it is assumed that β^p, β^n are equal for all time periods. Let $q_j = \beta^p q^p_j + \beta^n q^n_j$, let $Q_d = \beta^p Q^p_d + \beta^n Q^n_d$, and let $\tilde{Q}_d = \beta^p \tilde{Q}^p_d + \beta^n \tilde{Q}^n_d$. Then the demand can be rewritten to

$$\lambda_d = \bar{\lambda}_d + \tilde{Q}_{d-1}.$$

Continuously in time the hotel manager decides which subset $S \subset N$ to offer to arriving clients. According to this *offer set* S an arriving client purchases product j with probability $P_j(S)$ or declines from purchasing anything with probability $P_0(S)$. In accordance with Markov decision process literature a feasible solution π to the problem at hand is called a *policy*. Let Π be the set of all policies, where non-deterministic policies are allowed. The policy $\pi^* \in \Pi$ that optimises expected revenue is called an *optimal policy*.

Remark 1. Airlines, rental cars, and short-term storage space leases are other examples of application areas. In general, a company has certain types of *resources*, such as flight legs for airlines and usage days in car rentals and storage space leases, which in context of hotels is denoted by nights. For airlines it is more natural that resources have different capacities, since air planes on flight legs can have different volumes. A product is a combination of resources, for example an itinerary of multiple flights in airlines, or renting a car or storage space for multiple days.

2.2 Problem Formulation

Let $N(S_\pi(t)) \in \mathbb{N}^n$ be the stochastic process of the vector of purchases at time t under policy π, where $S_\pi(t) \subset N$ is the offer set corresponding to π. The problem statement is

[1] It is possible to let the arrival rate λ_d be time dependent. The analysis and results remain the same, but for ease of notation a constant arrival rate is used throughout this paper.

then given by

$$
\begin{aligned}
&\max_{\pi \in \Pi} E[\int_0^T r^\top N(S_\pi(t))dt] \\
&\text{s.t.} \int_0^T AN(S_\pi(t)) \leq C, \\
&\qquad S_\pi(t) \subset N.
\end{aligned} \tag{3}
$$

In order to solve this problem, time is discretised into T time units, such that the probability that more than one event occurs in one time unit is small. The probability that a customer arrives in time period t $(1 \leq t \leq T)$ is given by λ_t. Define the state space by $X \times Y$, where X is the set of all possible occupation scenarios, i.e., $X = \{x \in \mathbb{N}^m \,|x \leq C\}$, and Y is the set of all possible outcomes for reviews. Since there are only a finite number of decision moments, $X \times Y$ is finite, however large. Note that λ_t depends on the reviews y; therefore, denote the arrival probability by $\lambda_t(y)$, $y \in Y$. Denote the revenue to go at time t in state (x, y) by $V_t(x, y)$. Under these considerations an optimal policy can be found by solving the following Bellman equation:

$$
V_t(x,y) = \max_{S \subset N} \left\{ \lambda_t(y) \sum_{j \in S} P_j(S) \left[r_j + V_{t+1}(x + A_j, y + q_j) - V_{t+1}(x,y) \right] \right\} + V_{t+1}(x,y). \tag{4}
$$

The high dimensionality of the state space (which requires at least m dimensions to keep track of the consumed rooms per night) makes this stochastic problem intractable, similar to other choice-based network studies [Bront et al., 2009, Hosseinalifam et al., 2016, Liu and van Ryzin, 2008, Meissner and Strauss, 2012, Meissner et al., 2013, Strauss and Talluri, 2012]. Therefore, approximations have to be considered. The next sections are dedicated to performance measures, including an upper bound, and two heuristic approaches.

3 Deterministic Model

This section proposes a deterministic variant of the stochastic problem 3. The objective value is shown to be an upper bound to the stochastic problem. Furthermore, the upper bound is shown to be asymptotically tight.

3.1 CRLP Formulation

Consider the deterministic variant of the stochastic problem, where demand and reviews are deterministic and continuous variables. Under deterministic demand a mixed integer linear program formulation is proposed, called *choice-based review linear program (CRLP)*. In other studies concerning choice-based network RM a similar linear program is derived, called *choice-based deterministic linear program (CDLP)* [Gallego et al., 2004, Liu and van Ryzin, 2008]. However, the CRLP does not follow directly from the CDLP, since the demand parameter is not a constant any more, but depends on the past reviews. A crucial step in deriving the CRLP is the introduction of the decision variables $x(S, d)$, representing the total number of clients that are offered set S during period d. This contrasts earlier work [Gallego et al., 2004, Liu and van Ryzin, 2008], where $t(S)$ is used as a decision variable, representing the *time* that set S is offered rather than the total number of clients that was offered set S. While using $t(S)$ is an appropriate decision variable in their studies, a direct implementation to the review model leads to a non-linear program.

The objective of CRLP is the expected reward. A customer that is offered set $S \subset N$ leads to an expected reward of $\sum_{j \in S} r_j P_j(S)$. With $x(S,d)$ as decision variables, the objective of CRLP is then given by

$$\sum_{d=1}^{D} \sum_{S \subset N} x(S,d) \sum_{j \in S} P_j(S) r_j. \tag{5}$$

When set $S \subset N$ is offered to a customer, the expected resource consumption is equal to $AP(S)$. The capacity constraints are therefore given by

$$\sum_{d=1}^{D} \sum_{S \subset N} x(S,d) AP(S) \leq C. \tag{6}$$

To model the demand and time constraints, observe that the total demand over all sets $S \subset N$ offered during period d is upper bounded by the demand rate of period d times the length of period d:

$$\sum_{S \subset N} x(S,d) \leq (\bar{\lambda}_d + \tilde{Q}_{d-1})^+ (T_d - T_{d-1}). \tag{7}$$

Note that the demand rate is non-negative. For ease of notation, assume that $T_d - T_{d-1} = 1$ for all $1 \leq d \leq D$. Using the definition of discounted reviews this then leads to[2]

$$\sum_{S \subset N} x(S,d) \leq \left(\bar{\lambda}_d + \alpha^{d-1} Q_0 + \sum_{d'=1}^{d-1} \alpha^{d-d'-1} \sum_{j \in N_{d'}} \sum_{S \subset N} \sum_{d''=1}^{d'} x(S,d'') P_j(S) q_j \right)^+, \tag{8}$$

for all $1 \leq d \leq D$. For notational convenience, let $\tilde{\lambda}_d = \bar{\lambda}_d + \alpha^{d-1} Q_0$, and define $\mu \colon \{S \subset N\} \times \{1, \ldots, D\} \times \{1 \ldots, D\} \to \mathbb{R}$ by

$$\mu(S,d,d') := \sum_{j \in S} P_j(S) q_j \left(\sum_{d''=d'}^{d-1} \alpha^{d-d''-1} \mathbb{I}\{j \in N_{d''}\} \right), \tag{9}$$

for all $S \subset N$ and for all $1 \leq d, d' \leq D$. The constant $\mu(S,d,d')$ can be interpreted as the expected additional demand in period d per unit of demand in a previous period d' that set S was offered. Hence $\mu(S,d,d')x(S,d')$ is the expected additional demand for period d that follows from offering set S in period d'. The demand constraints (8) can now be rewritten in a more convenient form:

$$\sum_{S \subset N} x(S,d) \leq \left(\tilde{\lambda}_d + \sum_{S \subset N} \sum_{d'=1}^{d-1} x(S,d') \mu(S,d,d') \right)^+, \tag{10}$$

[2] Note that $\alpha^{d-d'}$ has to be replaced with $\alpha^{T_d - T_{d'}}$ in Equation (8) when $T_d - T_{d-1} = 1$ does not hold.

for all $1 \leq d \leq D$. The CRLP is then given by

$$\begin{aligned}
\max_{\substack{x(S,d):\\ S\subset N, 1\leq d\leq D}} & \sum_{d=1}^{D}\sum_{S\subset N} x(S,d)\sum_{j\in S} P_j(S)r_j \\
\text{s.t.} & \sum_{d=1}^{D}\sum_{S\subset N} x(S,d)AP(S) \leq C, \\
& \sum_{S\subset N} x(S,d) \leq \left(\tilde{\lambda}_d + \sum_{S\subset N}\sum_{d'=1}^{d-1} x(S,d')\mu(S,d,d')\right)^{+}.
\end{aligned} \tag{11}$$

The demand constraints contain a maximisation function, such that the problem can be solved as a mixed integer linear program (MILP). Mathematical software programs such as CPLEX and Gurobi are capable of handling quite large MILP instances.

3.2 Upper Bound and Asymptotic Optimality

Let V^{CRLP} be the optimal objective value of the CRLP (11) and let V^* be the optimal objective value of the stochastic problem (3). Proposition 3.1 shows that V^{CRLP} is an upper bound to V^* (similar to [Gallego et al., 2004, Liu and van Ryzin, 2008]).

Proposition 3.1. $V^{\text{CRLP}} \geq V^*$

Proof. Let π^* be an optimal policy of the stochastic problem (3) and let $S_{\pi^*}(t, \mathcal{F}_t)$ be the stochastic process of sets offered at time t under π^* and $\mathcal{F}_t$ the history of the system up to time t. Since π^* is feasible to (3), it holds in a path wise fashion that

$$\int_0^T AN(S_{\pi^*}(t, \mathcal{F}_t))dt \leq C,$$

and therefore the expectation is finite:

$$E[\int_0^T AN(S_{\pi^*}(t, \mathcal{F}_t))dt] \leq C < \infty.$$

Note that $AN(S(t))$ is non-negative, so Fubini's Theorem applies, with the following result:

$$\int_0^T AE[N(S_{\pi^*}(t, \mathcal{F}_t))]dt = E[\int_0^T AN(S_{\pi^*}(t, \mathcal{F}_t))dt] \leq C.$$

Next, define $x_{\pi^*}(S,d)$ by

$$x_{\pi^*}(S,d) = E[\lambda_d \int_{T_{d-1}}^{T_d} \mathbb{I}\{S_{\pi^*}(t, \mathcal{F}_t) = S\}dt],$$

for all $S \subset N$ and $1 \leq d \leq D$, with

$$\lambda_d = \left(\tilde{\lambda}_d + \sum_{S \subset N} \sum_{d'=1}^{d-1} x_{\pi^*}(S, d') \mu(S, d, d') \right)^+ .$$

The expected number of clients that purchase product j is equal to the summation over S of the expected number of sales of product j, given the number of clients that were exposed to set S:

$$\sum_{d=1}^{D} \sum_{S \subset N} x_{\pi^*}(S, d) P_j(S) = \int_0^T E[N_j(S_{\pi^*}(t, \mathcal{F}_t))] dt.$$

Therefore, the constraints of CRLP (11) are satisfied, and π^* is a feasible solution to CRLP (11). Furthermore, the objective values of the stochastic problem (3) and CRLP coincide:

$$V^* = E[\int_0^T r^\top N(S_{\pi^*}(t, \mathcal{F}_t)) dt] = \sum_{d=1}^{D} \sum_{S \subset N} x_{\pi^*}(S, d) \sum_{j \in S} P_j(S) r_j,$$

Since $x_{\pi^*}(S, d)$ is a feasible solution to CRLP, the corresponding objective value is bounded by the optimal objective value V^{CRLP}, which completes the proof. $\square$

Consider the k-scaled problem instance, where demand, capacity, and initial reviews Q_0^p and Q_0^n are scaled by a factor $k \in \mathbb{N}$, i.e., the demand rate equals $k\lambda$, the capacity vector equals kC, and the initial reviews are given by kQ_0^p and kQ_0^n. Let V_k^* and V_k^{CRLP} be the optimal objective value of the k-scaled stochastic problem and CRLP, respectively. As $k \to \infty$, the objective values of the stochastic and deterministic values converge to V^{CRLP}, which is shown in Proposition 3.2 below.

Proposition 3.2. $\lim\limits_{k \to \infty} \frac{1}{k} V_k^* = \lim\limits_{k \to \infty} \frac{1}{k} V_k^{\text{CRLP}} = V^{\text{CRLP}}$.

Proof. Let $x^*(S, d)$ be an optimal solution to the unscaled CRLP problem (11). The objective value of the k-scaled CRLP is equal to k times the objective value of the unscaled CRLP. Also, the constraints of the k-scaled CRLP equal k times the constraints of the unscaled CRLP. Therefore, $kx^*(S, d)$ is an optimal solution to the k-scaled CRLP with optimal value kV^{CRLP}, so the second equality above holds.

Now construct an optimal policy $\pi \in \Pi$ for the k-scaled stochastic problem from $x^*(S, d)$ as follows. In period d, offer set S a deterministic amount of time equal to

$$t_\pi(S, d) := x^*(S, d) / \lambda_d, \tag{12}$$

where λ_d is the deterministic demand rate given by

$$\lambda_d = \left(\tilde{\lambda}_d + \sum_{S \subset N} \sum_{d'=1}^{d-1} x(S, d') \mu(S, d, d') \right)^+ . \tag{13}$$

This is the time that set S is offered during period d in the (k-scaled) CRLP. The order that the sets are offered is arbitrary. Let $D^k(S,d,t)$ be the random vector of product demand under set S over t time units in the k-scaled stochastic problem. So, for product j and offer-set S in period d, $D^k_j(S,d,t_\pi(S,d)$ is Poisson distributed with parameters $x^*(S,d)P_j(S)$. Under π not all demand is accepted, only the demand below $kx^*(S,d)P(S)$ is accepted. Let $N_\pi(S,d)$ be the accepted demand according to policy π:

$$N_\pi(S,d) := \min\{D^k(S,d,t_\pi(S,d)), kx^*(S,d)P(S)\}. \tag{14}$$

Due to the demand constraint of the k-scaled problem of the CRLP it holds that

$$\begin{aligned}\sum_{d=1}^{D}\sum_{S\subset N} AN_\pi(S,d) &= \sum_{d=1}^{D}\sum_{S\subset N} A\min\{D^k(S,d,t_\pi(S,d)), kx^*(S,d)P(S)\}\\ &\leq kC,\end{aligned}$$

so π is an admissible policy for the k-scaled stochastic problem. The objective value equals

$$\sum_{d=1}^{D}\sum_{S\subset N} r^\top N_\pi(S,d) = \sum_{d=1}^{D}\sum_{S\subset N} r^\top \min\{D^k(S,d,t_\pi(S,d)), kx^*(S,d)P(S)\}. \tag{15}$$

Letting $k\to\infty$ this leads to

$$\lim_{k\to\infty}\frac{1}{k}\sum_{d=1}^{D}\sum_{S\subset N} r^\top \min\{D^k(S,d,t_\pi(S,d)), kx^*(S,d)P(S)\} \tag{16}$$

$$= \lim_{k\to\infty}\sum_{d=1}^{D}\sum_{S\subset N} r^\top \min\{\frac{1}{k}D^k(S,d,t_\pi(S,d)), x^*(S,d)P(S)\}. \tag{17}$$

For all $j\in N$, $D^k_j(S,d,t_\pi(S,d))$ has the same distribution as $\sum_{y=1}^{k} D_{j,y}(S,d,t_\pi(S,d))$, with $D_{j,y}(S,d,t_\pi(S,d)) \sim \text{Pois}(x^*(S,d)P_j(S))$ i.i.d. for all $1\leq y\leq k$. By the law of large numbers the sequence $\frac{1}{k}D^k(S,d,t_\pi(S,d))$ then converges to $x^*(S,d)P(S)$ a.s. as $k\to\infty$. Since the minimisation function is continuous, the continuous mapping theorem can be applied, which yields

$$\begin{aligned}&\lim_{k\to\infty}\sum_{d=1}^{D}\sum_{S\subset N} r^\top \min\{\frac{1}{k}D^k(S,d,t_\pi(S,d)), x^*(S,d)P(S)\}\\ &= \sum_{d=1}^{D}\sum_{S\subset N} r^\top \min\{x^*(S,d)P(S), x^*(S,d)P(S)\}\\ &= \sum_{d=1}^{D}\sum_{S\subset N} x^*(S,d)r^\top P(S)\\ &= V^{\text{CRLP}}.\end{aligned}$$

This completes the proof.

□

4 Solution Methods

In this section two solution methods are proposed to approximate the stochastic problem (3). The first method, called CRLP, uses the solution to CRLP (11) as a strategy. The second method is motivated by the fact that the demand rate depends on past reviews, which entails uncertainty. A robust optimisation formulation is proposed to deal with this uncertainty, with the goal of providing an improved solution method. Finally, this section is concluded with a discussion on some computational challenges and opportunities.

4.1 CRLP approximation

The first heuristic is a straightforward implementation of the outcome of CRLP. Let $x^*(S,d)$ be an optimal solution to CRLP. In term of time units, the optimal strategy $x^*(S,d)$ translates to offering set S for a total of $t(S,d) = x^*(S,d)/\lambda_d$ time units in period d. Since the strategy assumes deterministic demand, it dictates no specific order in which the sets are offered. Moreover, set S does not have to be offered continuously for $t(S,d)$ time units in period d: it may be offered a couple of different time segments within period d, as long as the total offer time equals $t(S,d)$.

However, in the stochastic model demand is stochastic while the capacity is limited. Each time a customer arrives, a purchase can only be accepted if the remaining capacity allows for it. By fixing the order in which sets S are offered in period d, for example by lexicographical ordering or ordering by expected revenue, products from sets S that are of higher order will be more likely to be purchased than lower order products, since by the time the lower order is reached, less capacity remains. This leads to a bias towards products of offer sets on top of the list and differs from core strategy of CRLP. An ordering might have a positive impact on performance, but this is not a straightforward process in general. Therefore, to overcome this problem and keep as close to the CRLP strategy as possible, a randomisation of offer sets is applied, in combination with splitting the period $t(S,d)$ in smaller parts. That is, the offer time $t(S,d)$ is split into K periods $t_k(S,d) = t(S,d)/K$ $(1 \le k \le K)$, such that the total time of offering set S in period d still equals $t(S,d)$. Then a random ordering is applied to all time segments $t_k(S,d)$ of all offer sets, per period d.

4.2 Robust CRLP

A crucial aspect of reviews is their impact on future demand. The CRLP assumes that the reviews are deterministic, and the CRLP approximation described in the previous section assumes that the demand rates are known. However, due to the stochasticity of the reviews, the actual demand rates may fluctuate. To overcome this problem, a robust version of the CRLP is proposed, where uncertainty is assumed in the outcome of the reviews. In particular, consider the demand constraint of CRLP (11):

$$\sum_{S \subset N} x(S,d) \le \left(\tilde{\lambda}_d + \sum_{S \subset N} \sum_{d'=1}^{d-1} x(S,d')\mu(S,d,d') \right)^+ \tag{18}$$

The demand in this constraint heavily depends on the outcome of the reviews, in the form of $\mu(S,d,d')$, which is stochastic. Small changes in the outcome of the reviews lead to different demand rates. To take this into account, assume that there is uncertainty in the parameters $\mu(S,d,d')$. For convenience, write $\mu(S,d,d')$ in terms of its nominal value $\bar{\mu}(S,d,d') \in \mathbb{R}$ and a *primitive factor* $\zeta(S,d,d') \in \mathbb{R}$:

$$\mu(S,d,d') = \bar{\mu}(S,d,d') + \zeta(S,d,d'). \tag{19}$$

The uncertain parameter $\zeta = \{\zeta(S,d,d')\}$ is assumed to lie in an uncertainty set

$$Z \subset \{S \subset N\} \times \{1,\dots,D\} \times \{1,\dots,D\}. \tag{20}$$

In [Ben-Tal et al., 2009] it is shown that the uncertainty can be approached constraint-wise. The robust formulation of constraint (18) can then be reformulated as

$$\sum_{S\subset N} x(S,d) \leq \left(\tilde{\lambda}_d + \sum_{S\subset N} \sum_{d'=1}^{d-1} x(S,d') \left[\bar{\mu}(S,d,d') + \zeta(S,d,d') \right] \right)^+, \tag{21}$$

for all $\zeta \in Z$.

The choice of the uncertainty set Z impacts the tractability of the program. Tractable formulations of the robust counterpart for several standard uncertainty regions are provided in [Ben-Tal et al., 2009]. Although the robust counterparts of those standard uncertainty regions are denoted as tractable, some still involve solving non-linear programs, and often the number of constraints increases. In case of the CRLP, tractability is already an issue. Under these deliberations, consider the interval/box uncertainty set Z_∞, given by

$$Z_\infty = \{\zeta \mid \|\zeta\| \leq \rho\}, \tag{22}$$

which leads to the following robust counterpart (see [Ben-Tal et al., 2009])[3]:

$$\sum_{S\subset N} x(S,d) \leq \left(\tilde{\lambda}_d + \sum_{S\subset N} \sum_{d'=1}^{d-1} x(S,d') \left[\bar{\mu}(S,d,d') - \rho \right] \right)^+. \tag{23}$$

The advantage of this uncertainty region is that the constraints remain linear and the number of constraints does not increase. However, using the same ρ for all variables is very conservative. Therefore the author of this paper proposes the following uncertainty set:

$$Z = \{\zeta \mid |\zeta(S,d,d')| \leq \rho(S,d,d')\}, \tag{24}$$

with $\rho(S,d,d') > 0$, $S \subset N$, $1 \leq d, d', D$. Under this uncertainty region the robust counterpart becomes

$$\sum_{S\subset N} x(S,d) \leq \left(\tilde{\lambda}_d + \sum_{S\subset N} \sum_{d'=1}^{d-1} x(S,d') \left[\bar{\mu}(S,d,d') - \rho(S,d,d') \right] \right)^+. \tag{25}$$

This adaptation of the interval/box uncertainty region is conservative but has two main advantages. First, it leads to a MILP which is tractable in the case of CRLP: no new integer variables are added and the number of constraints of the robust CRLP remains the same. Second, the nominal values $\bar{\mu}(S,d,d')$ may very from each other in orders of magnitude, but Z can take this into account by setting the bounds $\rho(S,d,d')$ relatively to the nominal value $\bar{\mu}(S,d,d')$.

[3] Note that $|x(S,d)| = x(S,d)$, since $x(S,d) \geq 0$.

4.3 Computational Challenges

The CRLP has $D2^n$ decision variables and $D + m$ constraints. With an exponential number of decision variables this problem is intractable, even for modest sizes. However, certain conditions on demand and branch-and-price techniques can be used to attempt to solve the problem. This paragraph examines both techniques in context of CRLP (11).

4.3.1 Segmentation under Disjoint Consideration Sets

An important step in improving the tractability of CRLP is to make assumptions on that demand that reduce the amount of decision variables. These assumption may follow naturally from the problem at hand, for example from segmentation of customers. Segmentation is an important aspect of revenue management and one of the first steps of a revenue management implementation ([Talluri and Van Ryzin, 2004], page 579-585). Assume that customers are partitioned into $|L| \in \mathbb{N}$ segments, where L is the set of segments and each segment $l \in L$ has its own characteristics and preferences. According to these characteristics and preferences, the hotel can target a segment $l \in L$ by offering them certain products S_l. Customers may be segmented in such a way that products can be offered solely to one segment, without being exposed to another, through different booking channels. For example, transient clients might use booking websites, while group reservations are made through agencies directly with the hotel; and [Sierag et al., 2016] show that business and leisure customers can be segmented by day of the week and time of the day: business clients tend to make a reservation during weekdays between 8:00 and 17:00, while leisure clients tend to make reservations during weekdays from 17:00 and in the weekend.

Now the assumption that reduces the number of decision variables is that segments have disjoint consideration sets N_l, $l \in L$. That is, a customer from segment $l \in L$ only considers a subset $N_l \subset N$ of the products, with $N_l \cap N_k = \emptyset$ for all $l, k \in L$. Although room consumption may be overlapping, the decision variables $x(S, d)$, for all $S \subset N$ and for all $1 \leq d \leq D$, can be replaced by $x(S_l, d)$, for all $S_l \subset N_l$, $l \in L$, and for all $1 \leq d \leq D$. This reduces the number of decision variables from $D2^n$ to $D \sum_{l \in L} 2^{|N_l|}$, a significant reduction. If the consideration sets N_l are of reasonable size, the number of decision variables can be contained and CRLP is tractable.

In other choice-based revenue management models, such as [Kunnumkal and Topaloglu, 2008, Liu and van Ryzin, 2008, Meissner and Strauss, 2012, Vossen and Zhang, 2015], a special case of segmentation under disjoint consideration sets is discussed and used to validate their models. In their work, the multinomial logit model is used as a choice model, and to build on their work the same choice model is used in the numerical section 5 of this paper. However, we stress that the reduction of decision variables holds for any choice model in combination with segmentation under disjoint consideration sets.

4.3.2 Branch-and-Price

When the number of decision variables of CRLP is too large, branch-and-price techniques can be used to attempt to solve CRLP. Branch-and-price is a column generation technique combined with branching for MILPs and is strongly related to branch-and-cut methods. See [Barnhart et al., 1998] for a general discussion of branch-and-price and [Cóccola et al., 2015] for an application of branch-and-price to a ship routing and scheduling problem. In the following, a sketch of a branch-and-price application to CRLP is given.

The branch-and-price algorithm consists of two aspects: 1) a branching tree to deal with the integer variables, and 2) a column generation procedure, executed in each node of the tree, to determine whether to branch or to stop, according to some decision rules. The column generation procedure gives a *local upper bound (LUB)* for each node in the branching tree. If the strategy of this LUB is feasible to CRLP, then the LUB is compared with the *global lower bound (GLB)*, the best feasible solution to CRLP found so far. If the LUB is lower than the GLB (whether it is feasible to CRLP or not), then no optimal solution will be found in this branch and it is explored no more. If the LUB is not feasible to CRLP and larger than the GLB, then this node is explored further by splitting it into two child nodes according to some decision rule.

In particular, the column generation procedure in a node of the branching tree is as follows. Instead of solving the CRLP *master problem (MP)* (11), a subset $N_d \subset N$, $d = 1, \dots, D$, of offer sets is considered as decision variables (columns of CRLP). This *reduced master problem (RMP)* is tractable when $|N_d|$ is substantially smaller than $|N|$, for all $d = 1, \dots, D$. The LP relaxation of the RMP leads to dual variables $\pi \in \mathbb{R}^m$ and $\rho \in \mathbb{R}^D$, corresponding to the capacity constraints and demand constraints, respectively. Next, consider the reduced costs, that follow from the dual solution of the relaxed RMP, of all columns of the MP that are not in RMP. If any of those columns has positive reduced costs, one of those columns is selected and added to the RMP. To determine which column has positive reduced costs and can be added to the RMP, the following column generation *sub-problem* has to be solved:

$$\max_{\substack{S \subset N \\ 1 \leq d \leq D}} \left\{ (r^\top - \pi^\top A) P(S) - \rho_d + \sum_{d'=d+1}^{D} \rho_{d'} \mu(S, d', d) \right\}. \tag{26}$$

If the optimal value of the sub-problem (26) is negative (i.e., if there is no column with positive reduced costs) and the relaxed RMP strategy is feasible to RMP (i.e, the relaxed integer variables are integers), then the strategy/LUB is compared with the GLB. If it is smaller than GLB, the exploration of this branch ends here.

Otherwise, if the optimal value of the sub-problem (26) is negative and the relaxed RMP solution is not feasible to RMP, branching is applied to the integer variables. If the LUB of the node is greater than GLB, then, according to some decision rule, an integer variable is selected to construct the next step in the branching tree. Otherwise the search in this branch ends here. This procedure is continued until an optimal strategy is found. The process always converges since the branching tree is finite.

Other papers on choice-based network revenue management also discuss column generation techniques [Gallego et al., 2004, Liu and van Ryzin, 2008]. However, they consider CDLP, which a linear program without any integer variables. Those procedures cannot be applied to CRLP, but the branch-and-price procedure described in this section can.

5 Numerical Results

This section provides numerical experiments to illustrate the model and performance of the solution methods. The first experiment focuses on the performance of the robust solution method under different loads; the second experiment focuses on the weight of review probabilities; the third experiment considers different hotel sizes; and the fourth and final experiment shows the performance under estimation errors of the demand parameters.

Suppose the manager wants to find a strategy for his hotel of size $C \in \mathbb{N}$ over an 8 week period. After each week reviews are released, so $D = 8$. Assume that rooms are sold over a 90 day booking horizon. For simplicity only two resource types per week are used, by grouping weekdays together: resource type 1 represents Monday-Friday, resource 2 represents Friday-Monday. See Figure 1 for an overview.

<table>
<tr><td colspan="7">Product type 3: Week</td></tr>
<tr><td colspan="4">Product type 1: Weekdays</td><td colspan="3">Product type 2: Weekend</td></tr>
<tr><td>Mon</td><td>Tue</td><td>Wed</td><td>Thu</td><td>Fri</td><td>Sat</td><td>Sun</td></tr>
</table>

Figure 1: The three different product types per week.

On these resources, three product types are constructed: product type 1 consumes one unit of resource type 1, representing a midweek stay; product type 2 consumes one unit of resource type 2, representing a weekend stay; product type 3 consumes one unit of resource type 1 and one unit of resource type 2, representing a whole week stay. Each product type has three price levels, see Table 1.

		Product		
Product type		1	2	3
1	(weekdays)	360	480	600
2	(weekend)	250	340	400
3	(week)	500	750	900

Table 1: Prices per product per product type.

An arriving customer is only interested in one product type in one particular arrival week d, $1 \leq d \leq 8$. Hence customer can be segmented such that their consideration set is restricted to the particular arrival week and product type. The base arrival rate of customers for week d equals $\bar{\lambda}_d = 2C$. Within one week, the probability that an arriving customer is interested in product type 1, 2, and 3 is equal to 1/2, 1/3, and 1/6, respectively. Furthermore, assume that customers of each segment select a product based on the multinomial logit model, where the MNL weights are given by $(10, 7, 3, 4)$ (the last entry represents the no-purchase option).

Regarding the reviews, assume that the positive and negative review probabilities equal $q^p = (0.1, 0.05, 0.01)$ and $q^n = (0.01, 0.02, 0.05)$, respectively, for all product types. Let $\alpha = 0.9$ be the discount parameter and let $\beta = (1, 1.5)$ be the parameter that measures the impact of reviews on demand. The initial reviews are set to zero.

The solution methods that are used in the examples are CRLP and the robust CRLP. The uncertainty parameters $\rho(S, d)$ are defined by

$$\rho(S, d) = \rho|\mu(S, d)|, \tag{27}$$

where $\rho \in \mathbb{R}^+$ is fixed for all $S \subset N$ and for all $1 \leq d \leq D$. The robust CRLP method under parameter $\rho \in \mathbb{R}^+$ is denoted by CRLP$_\rho$. CDLP is used as a benchmark. The primary measure of performance is the expected revenue. A secondary objective is the variation of revenue in terms of the coefficient of variation c_v, which can measure the robustness of the heuristic.

To analyse the impact of different problem sizes some scaling parameters are used for hotel size, demand load, and review probabilities. These parameters are specified in the appropriate examples.

Simulations are used to approximate the expected revenues of the different heuristics. The errors are within 0.5% of the stated values, with 95% confidence.

5.1 Demand Load

The first experiment validates the performance of the different heuristics under various demand loads. The hotel size is set to $C = 200$ and demand is multiplied by a *load factor* $l_f \in \{0.6, 0.8, 1, 1.2, 1.4\}$. The applied heuristics are CRLP and CRLP$_\rho$ for $\rho \in \{0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1\}$. The results are presented in Table 2.

The solution methods behave differently under the different demand loads. Surprisingly, an increase in demand load brings the optimality gaps of CRLP and CDLP closer together. One would assume that under higher loads the effect of reviews is much more evident, which translates to a better performance of CRLP. A plausible explanation is that the strategies of CRLP and CDLP are similar when the demand load is increased. If demand is high, then it makes less sense to sacrifice revenue now in order to get better reviews, and higher demand, in the future: there will be enough demand anyway. However, when demand is low, then it is worth sacrificing some revenue now in order to get higher demand in the future.

The robust solution methods perform quite well, in some cases even better than CRLP. Also, the coefficient of variation of CRLP$_\rho$ is generally lower than of CRLP. An exception is at $l_f = 0.6$, though the difference is small. Based on these observations, CRLP$_{0.5}$ is selected to be used in the other examples: it provides a lower c_v than CRLP, indicating that the solution is more stable; and the expected revenue does not deviate that much from CRLP, and for $l_f = 1$ it is equal (in the remaining example the demand load is fixed to $l_f = 1$).

5.2 Weight of Review Probabilities

This sections investigates the impact of the weight of the review probabilities. In an environment with $C = 200$ rooms and no scaled load (see the previous experiment), the review probabilities are multiplied with a weight $w \in \{0.01, 0.1, 0.2, 0.5, 1, 2\}$. A low

l_f	Method	Revenue	c_v (%)	% Opt. gap
	UB	805698	0.00	0.00
	CRLP	803539	5.75	0.27
	CDLP	688235	5.73	14.58
0.6	0.05	803353	5.72	0.29
	0.1	802974	5.74	0.34
	0.2	803626	5.85	0.26
	0.5	798323	5.84	0.92
	1	787477	5.98	2.26
	UB	1060989	0.00	0.00
	CRLP	1037750	3.53	2.19
	CDLP	916199	4.94	13.65
0.8	0.05	1037682	3.43	2.20
	0.1	1037283	3.28	2.23
	0.2	1035870	3.26	2.37
	0.5	1027387	3.04	3.17
	1	1032296	3.79	2.70
	UB	1220873	0.00	0.00
	CRLP	1187402	2.90	2.74
	CDLP	1136718	3.72	6.89
1	0.05	1188385	2.83	2.66
	0.1	1188001	2.76	2.69
	0.2	1190194	2.65	2.51
	0.5	1187481	2.24	2.74
	1	1175339	1.99	3.73
	UB	1326409	0.00	0.00
	CRLP	1282403	2.34	3.32
	CDLP	1251263	2.95	5.67
1.2	0.05	1283714	2.25	3.22
	0.1	1283909	2.13	3.20
	0.2	1284633	2.15	3.15
	0.5	1286872	1.95	2.98
	1	1282403	1.71	3.32
	UB	1403950	0.00	0.00
	CRLP	1355459	2.13	3.45
	CDLP	1339087	2.68	4.62
1.4	0.05	1356163	1.99	3.40
	0.1	1357059	1.98	3.34
	0.2	1356214	1.80	3.40
	0.5	1352999	1.68	3.63
	1	1345883	1.73	4.14

Table 2: Performance of different heuristics under various demand loads l_f.

w	UB	CRLP Rev	c_v	%Gap	CDLP Rev	c_v	%Gap	$\text{CRLP}_{0.5}$ Rev	c_v	%Gap
0.01	1165145	1154996	1.87	0.87	1154996	1.87	0.87	1154996	1.87	0.87
0.1	1170226	1148537	1.84	1.85	1153348	2.09	1.44	1149764	1.91	1.75
0.2	1178102	1153052	2.00	2.13	1151556	2.27	2.25	1153876	1.97	2.06
0.5	1197396	1168466	2.36	2.42	1144965	2.96	4.38	1165078	2.17	2.70
1	1220873	1187402	2.90	2.74	1136718	3.72	6.89	1187481	2.24	2.74
2	1281146	1232147	3.09	3.82	1115901	5.02	12.90	1223037	1.84	4.54

Table 3: Performance of different heuristics under various weights w of review probabilities.

weight implies a small effect of review probabilities, while a high weight implies a large effect of review probabilities. The results are presented in Table 3.

The results show that the weight of the review probabilities impacts the performance. For the low values of $w = 0.01$ the performances of all heuristics give the same results. CRLP outperforms CDLP in all other cases, with the surprising exception of $w = 0.1$. $\text{CRLP}_{0.5}$ performs better than CRLP for small weights, but when the weight increases CRLP performs better in terms of expected revenue and worse in terms of variation.

5.3 Hotel size

In this experiment the hotel size is varied as $C \in \{50, 100, 200, 500, 1000\}$, while the other parameters remain fixed. The results are presented in Table 4.

C	UB	CRLP Rev	c_v (%)	%Gap	CDLP Rev	c_v (%)	%Gap	$\text{CRLP}_{0.5}$ Rev	c_v (%)	%Gap
50	305218	287855	5.35	5.69	278054	6.44	8.90	288752	5.07	5.39
100	610436	586188	4.11	3.97	563560	4.98	7.68	587729	3.42	3.72
200	1220873	1187402	2.90	2.74	1136718	3.72	6.89	1187481	2.24	2.74
500	3052182	3000490	1.84	1.69	2859800	2.58	6.30	2991797	1.22	1.98
1000	6104363	6030491	1.28	1.21	5731378	1.88	6.11	5999069	0.75	1.72

Table 4: Performance of heuristics under different hotel sizes C.

There are two main observations to be made, apart from the fact that CRLP and $\text{CRLP}_{0.5}$ outperform CDLP for all hotel sizes, both in terms of expected revenue (by 3.5-5.2 percent) and variation. First, the method that performs best depends on the hotel size: for small hotels, with $C = 50$ and $C = 100$, $\text{CRLP}_{0.5}$ outperforms CRLP in terms of both revenue and variation. On the other hand, for large hotels, with $c = 500$ and $C = 1000$, CRLP provides higher expected revenue but $\text{CRLP}_{0.5}$ provides lower variation. For small hotels it is therefore beneficial to use the robust CRLP method, while large hotels need to decide on the trade-off between revenue and variation. As a large hotel can bare more risk due to its large volume, the CRLP method will lead to higher revenues in the long run.

A second observation is that the optimality gap and variation decrease as the hotels size increases. There are two forces at work that could explain this behaviour. First, by Propositions 3.1 and 3.2, the upper bound is larger than the optimal value, and converges to the optimal value as the hotel size and demand increase. Second, the CRLP heuristic might perform better as hotel size and demand increase. In both cases the difference between the upper bound and CRLP gets tighter.

5.4 Estimation Error

In this experiment the performance of the heuristics is measured under estimation errors of demand. The strategies of the heuristics are evaluated using the current parameters, but the actual demand is assumed to deviate from $\bar{\lambda}_d$ by a factor of $u \in \mathbb{R}^+$. In each simulation, u is drawn from a uniform distribution: $u \in \text{Unif}[1-\bar{u}, 1+\bar{u}]$, with $\bar{u} \in \{0.01, 0.02, 0.05, 0.1, 0.2, 0.5\}$. The results are presented in Table 5.

	CRLP			CDLP			$\text{CRLP}_{0.5}$		
$\bar{u}$	Rev	c_v (%)	%Loss	Rev	c_v (%)	%Loss	Rev	c_v (%)	%Loss
∞	1187200	2.89	0.00	1136084	3.78	4.31	1187561	2.21	-0.03
0.01	1187586	2.84	-0.03	1135729	3.76	4.34	1187495	2.26	-0.02
0.02	1187304	2.95	-0.01	1136061	3.84	4.31	1187025	2.40	0.01
0.05	1184460	3.44	0.23	1134494	4.43	4.44	1186081	2.77	0.09
0.1	1178394	4.82	0.74	1131404	5.88	4.70	1181261	3.97	0.50
0.2	1158335	8.49	2.43	1117748	9.67	5.85	1162424	7.45	2.09
0.5	1078115	21.06	9.19	1059288	22.12	10.77	1090038	19.90	8.18

Table 5: Performance of heuristics under estimation errors.

The results show that CRLP is quite robust in the sense that the performance does not suffer too much in presence of estimation errors. For estimation errors of up to 5% the revenue loss compared to perfect knowledge does not deviate too much, and in fact increases a little bit (although this might be caused by simulation errors). For all values of $\bar{u}$ the variation is lowest for CRLP_ρ. This is expected, as the robust solution method takes uncertainty into account. However, when the uncertainty is too big, with $\bar{u} = 0.5$, all methods do not perform well.

6 Conclusion

Online reviews have a big impact on revenue. This paper introduces a choice-based network revenue management model that incorporates reviews in a feedback mechanism: on the one hand demand is impacted by reviews, and on the other hand, reviews depend on the price/quality perception of customers. Including reviews in the decision process can lead to a significant increase in revenue.

The full stochastic problem cannot be evaluated for practical purposes, a common problem in choice-based network RM, because of the network structure and because the problem has to keep track of the reviews. To this extent a deterministic variant of the problem is proposed. The deterministic variant serves as an upper bound to the stochastic problem (Proposition 3.1), and converges to the optimal stochastic value when demand and capacity are scaled (Proposition 3.2).

Two heuristics are proposed to solve the problem: one based on the deterministic variant and a robust optimisation approach, where the outcome of reviews is assumed to be uncertain. Both heuristics show a significant improvement of 3.5-5.2 percent over the benchmark, which does not take into account reviews (see Section 5). Two insightful results from the numerics in Section 5 are 1) that considering reviews in the decision process has more impact when demand is low compared to when demand is high; and 2) that small hotels are more effected by the review feedback mechanism than larger hotels. Two other core advantages make the heuristics an effective tool for practitioners: both heuristics can be evaluated efficiently, and the robust solution method reduces the risk that comes with estimation errors.

The remainder of this section briefly addresses two important aspects of choice-based network RM under reviews. First, opportunities and challenges of various application areas are discussed. Second, as a topic for further research, the value of dynamic strategies is highlighted, as well as challenges to acquire such strategies.

6.1 Application Areas

The choice-based review model can be applied in many relevant application areas. Hotels, airlines, travel agencies, rental cars, and short-term storage space leases have a network structure and sales are highly influenced by recent reviews. Websites like booking.com, hotels.com, tripadvisor, airlinequality.com, rentalcars.com, and yelp assist customers in their decision process by offering reviews of the various options of companies and products.

The effect of reviews on demand differs per industry. In some industries, like hotels, the reviews impacts the demand of the *individual property* more than the brand itself. This is due to the fact that ratings differ per property of the same branch, and for a property only corresponding reviews are listed. In other industries, on the other hand, like airlines, *brands* are rated rather than individual flight legs.[4] The effect of a review of a flight leg therefore impacts the whole fleet. One way to deal with this is joining the whole fleet together in an optimisation problem, such that the effects of reviews over multiple flight legs is accounted for. However, this leads to intractability issues of biblical proportions. A tractable approach would be to optimise local fleets, where the effect of reviews from flight legs outside the local fleet are accounted for by forecasts.

Applications of reviews to RM that where a network structure is absent, or where booking horizons don't overlap, are treated in [Sierag and Roijers, 2016].

6.2 Further Research

As this is a pioneering study on choice-based network RM under reviews, investigating improved solution methods to tighten the optimality gap is a promising future direction. In particular, the heuristics described in this paper are of static rather than dynamic nature. However, other choice-based revenue management studies have shown an improvement in performance when dynamic strategies are used rather than static strategies (e.g., see [Liu and van Ryzin, 2008, Maglaras and Meissner, 2006]). However, most studies use only the current state of reservations to decide on an offer set, and clearly not the current state of reviews. This is exactly why a straightforward implementation of such

[4] As a hybrid, car rental sites rate the brands, but tend to show reviews related to the pickup location.

strategy most likely will fail: it does not take into account the crucial effect of reviews, leading to a downwards spiral of decreasing reviews and decreasing demand.

Decomposition methods, like in [Liu and van Ryzin, 2008] and [Talluri and Van Ryzin, 2004], page 100-108, solve the problem of the huge state space of the full dynamic problem by solving a number of single-night stay sub-problems. However, in review context, these sub-problems would still be intractable if reviews are to be taken into account. The main challenge is to develop a novel method that incorporates the reviews in the decision process without blowing up the state space.

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